AN ANALYSIS OF SOME
MACHINE TOOL REPLACEMENT FORMULAS

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ABSTRACT

American Industry is faced with the never ending problem of deciding when to replace machine tools. Over the years a number of machine tool replacement formulas have been developed to aid in the solution of the problem.

The purpose of this thesis is to determine:

1. The differences in the methods used and the factors considered by the various formulas in solving a replacement problem.
2. The effect of errors in estimation on the results given by a particular formula.

Six formulas are analyzed; they are the Annual Cost, the National Machine Tool Builders Association (NMTBA), the Machinery and Allied Products Institute (MAPI), Norton's, Discounted Cash Flow, and Rule of Thumb formulas. A machine tool replacement problem is solved by each of the formulas and the answers yielded are analyzed. Then errors are introduced into the problem and the answers are analyzed again to determine the effect of the errors on the answers.

Some of the factors that are considered by the formulas are depreciation, estimated salvage value, interest rate, etc. The list is too long to present here. All of the formulas do not use the same factors and very often when a factor is considered by two formulas, it is not considered in the same way.

The formulas are generally designed to make a comparison between alternative machines, the answer to the comparison being given as an annual saving. Usually no two formulas will give the same answer to

a specific problem.

In the older formulas a calculation is generally made to find the savings that will accrue in the year the replacement is made and then it is assumed that this saving will be realized for each year of life for the new machine. The more recent formulas, notably the MAPI formula, tend to give more attention to the operating conditions that are anticipated in the future. The future annual savings are reduced by considering both deterioration on the machine and the greater efficiency of new machines.

Many of the formulas require estimations of service life, terminal salvage value, and interest rate. Errors in estimating these factors affect the answers yielded by the formulas. For a machine tool replacement problem considered in this paper, the answer to the MAPI formula was found to be affected more by variations in these three factors than the answers to the other formulas tested.

Machine tool replacement formulas should never be used without thorough study because of the wide variations in answers that can be expected and because of the effect that errors in estimation have on the answers. The derivation of the formula and its method of handling the various factors should be understood so that the result of the formula can be evaluated properly by the executive who must make the final replacement decision.

AN ANALYSIS OF SOME
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INTRODUCTION AND PURPOSE

In conducting a business enterprise, the management is required to make numerous decisions. The decisions are always between alternatives, to buy or not to buy, to sell or not to sell, to produce a new line of goods or to continue producing the old line. The basis of these decisions is to maximize profits and minimize losses, although there are times when the governing factor in the decision may be to increase the prestige of the company or put the company in a better position in the industry. Even these latter decisions have greater future profits as the motivating influence. In order to make these decisions wisely, it is necessary for the management to gather data on all the factors that will influence or be influenced by the decision. These data should be assigned monetary values if at all possible. The data must then be carefully weighed and the alternative that yields the highest profit should be selected.

Unfortunately, all business decisions are not made in this manner. In many cases "hunch decisions" are made without considering all of the factors that will be affected by the decision. In many cases, rules of thumb, considering just a few of the factors involved, determine whether a proposal will be accepted or rejected. The use of rules of thumb are widely used by American Industry in making

machine tool replacement decisions.[1]

The decision to keep an old machine tool or to purchase a new one should be motivated by the desire to maximize profit or minimize loss, just like any other business decision. All of the factors that will be affected by the decision should be carefully weighed to see if replacement will yield a greater profit than will be realized by continued use of the present machine. The manner in which the factors involved should be weighed in making a machine tool replacement decision is not generally agreed upon.[2] The consequence has been that a rather large number of machine tool replacement formulas have been proposed to aid management in making machine tool replacement decisions. The results given by these formulas usually vary widely and so their value to management in making decisions is doubtful.[3]

The purpose of this thesis is an investigation of some of the more widely used formulas to determine what factors are considered and how the factors influence the result. The limitations and strengths of individual formulas are pointed out and suggestions are made for when to use a particular formula in a particular replacement problem.

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1. Terborgh, G., Dynamic Equipment Policy, New York: McGraw-Hill, 1949, p. 12.

2. Dana, F. C., "Replacement of Equipment," Journal of Engineering Education, March, 1948, p. 447.

3. Sharpe, H. D., Jr., "Replacement Formulas; Are They a Help or a Headache?", Tool Engineer, August, 1953, p. 43.

CURRENT PRACTICE

The methods used by American Industry to determine when to replace equipment are varied and apparently inadequate. For instance, in 1953, $969,000,000 was wasted by metal working industries in the United States. This huge sum was paid for direct labor alone on machine tools that cannot meet today's productivity standards. Machine tools designed today are forty percent more productive than the machine tools built a decade ago and ninety-five percent of the machines used by the metal working industries are over ten years old or are of designs that are over ten years old.[4]

The methods used by American Industry in handling machine tool replacement problems are revealed by a number of surveys taken in the last five years.

In one survey conducted in 1948 by MAPI, two hundred manufacturers of capital equipment reported:[5]

> Scarcely more than one quarter of the companies responding have an engineer who specializes in replacement studies and it is evident that the customers of these companies are little, if any, better in this respect. Only one third make any regular periodic review of their equipment situation for the purpose of improvement or modernization. Again, only one third make any attempt to budget equipment expenditures ahead. While more than half keep repair and maintenance records for each unit of equipment, the proportion keeping other records relevant to replacement analysis falls back to one third.

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4. "American Machinist Mid-Century Inventory," <u>American Machinist</u>, Mid-November, 1953, vol. 97, p. B.

5. <u>MAPI Survey of Replacement and Depreciation Policies</u>, Machinery and Allied Products Institute, Washington, October, 1948, Bulletin No. 2119, pp. 3-11.

With respect to the origination of recommendations for re-equipment, it appears that these come prevailingly from the regular operating executives such as foremen, master mechanics, superintendents, works managers, and department heads, while the final decision rests, as a rule, with the general officers, including, in a substantial proportion of the cases, the board of directors.

. . . An important contributing factor to backward and irregular mechanization is lack of capital. . . . This widespread dependence of equipment policy on the state of the treasury is indicated in the replies to the question, "To what extent is the customer's current liquid position a factor in his decision to buy or not to buy?", to which 82 per cent replied that the liquid position is important, if not, indeed, absolutely determining. This report as to the behavior of customers is reinforced by the admission of 43 per cent of the respondents that their own requirements for replaceability vary with their liquid position.

There is, on the whole, a remarkable uniformity in the replies to the section of the questionnaire dealing with depreciation policy, and it appears almost certain, therefore, that they are satisfactorily representative of capital goods manufacturers in general. The typical company has no regular procedure for reappraising the remaining life of equipment and revising its depreciation rates accordingly. It does not earmark accruals for the purchase of new equipment. It does not tie depreciation rates to variations in use. It takes the same rates for income tax and for book purposes. It sets up its composite accounts by year of acquisition.

Reflecting in part the tendency of the Bureau of Internal Revenue to standardize rates for tax purposes, there is a fairly high degree of concentration in the rates reported. As to buildings, 80 per cent of the companies responding show average rates between 2 and 3 per cent inclusive. For equipment, 74 per cent report averages falling between 5 and 8 per cent inclusive. More interesting than these actual rates are the rates that would be taken, for tax purposes, if the respondents had full discretion to name their own. For buildings, 87 per cent would take rates of 5 per cent or under, while for equipment, 73 per cent would take 10 per cent or less. Nearly half would take exactly 10 per cent.

Another survey of machine tool manufacturers was conducted by <u>Steel</u> magazine in 1953.[6] Responses to the survey came from 2,104 companies employing 20 or more employees.

The answers to the question, "Who decides machine tool purchases?"

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6. "Special Report," <u>Steel</u>, November 16, 1953, vol. 133, pp. 7-9.

in general bear out the findings of the MAPI survey. They found that machine tool purchases are not generally decided upon by one individual, the decision is made by group action. The president has a greater influence in deciding machine tool purchases in smaller companies than in large companies. Companies of 50 to 99 employees report the president as the prime buying influence in 73 percent of the cases. In companies of over one thousand employees, only 24 percent of the companies report him as a prime buying influence. The superintendent, works manager, tool engineer, master mechanic, and foreman have a greater influence in larger companies than in smaller companies. For example, the tool engineer exerts a strong influence in eight percent of the companies employing 50 to 99 employees as compared to 48 percent of the companies employing more than one thousand employees.

Table 1 contains a compilation of the answers to the question, "Who decides machine tool replacements?"

Only one quarter of all the companies surveyed had a planned replacement program. For companies of over one thousand employees, one-half had planned programs. Most of the companies felt a need for a planned program but claimed that they were unable to set up a good program because of insufficient funds to purchase equipment because of heavy tax burden, amortization periods were too long, and inflation had lifted prices for current machine tools well above the cost of the original machines.

Less than half of the companies earmark funds, other than depreciation allowances, for new equipment purchases.

Approximately 79 percent of the companies in the survey would

Table 1

Who Decides Machine Tool Replacements

Individual	Percentage of Companies
President	56.7
Superintendent	54.2
Works Manager	39.3
Vice President	37.4
General Manager	35.9
Purchasing Agent	30.8
Chief Engineer	26.1
Foreman	25.2
Tool Engineer	21.6
Master Mechanic	14.8
Treasurer	11.2
Secretary	4.3
Machine Operator	2.2

like to have the option of selecting their own amortization period for machine tools. The periods desired are shown in Table 2 as are periods chosen by companies in the MAPI survey of 1948.

Another trade magazine, Modern Materials Handling, reports on an equipment replacement policy survey taken in 1952.[7]

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7. Bright, James R., "What Is a Sound Equipment Replacement Policy?", Modern Materials Handling, October, 1952, VII, 10, p. 57.

Table 2

Amortization Periods Desired by Companies

Steel Survey		MAPI Survey	
Amortization Period	Percentage of Companies		Amortization Period
Under 3 years	1.5	8	5 years or less
3 to 7 years	26.0	9	6 to 9 years
8 to 12 years	59.9	48	10 years
13 to 17 years	8.3	25	Over 10 years
Over 17 years	4.3	10	Varying

Less than one firm in 40 has a definite replacement policy. Out of every 10 firms you'll find at least 7 completely different procedures for replacement of equipment.

About 50% of the men in charge of studying equipment requirements and developing materials handling programs volunteered the comment that they had no particular equipment replacement program, and their own thinking was very fuzzy on what kind of a program they should have.

There have been a number of articles published in the trade periodicals advancing reasons for the present industrial practice of keeping obsolete machines in service. G. F. Sullivan[8] offers three reasons for continuance: (1) Usual depreciation reserves for replacing machine tools are too small because of the post-war price increases, (2) federal tax policies discourage machine tool replacement "until the last dog is hung," (3) industry in general has a hit or miss approach to

8. Sullivan, G. F., "Depreciation Rules Curb Industrial Progress," Iron Age, April 27, 1950, pp. 79-83.

the problem. E. M. Hicks[9] gives essentially the same three reasons for the condition: (1) Industry has been using a haphazard rule of thumb approach to the problem of machine tool replacement, (2) there is at present a false impression of what is a conservative replacement policy, (3) the depreciation allowed on machine tools by the Treasury Department hinders a progressive replacement policy.

The reasons given by Hicks and Sullivan can be placed under two broad classifications; those dealing with depreciation policies, and those involving machine tool replacement policies.

This thesis is concerned chiefly with machine tool replacement formulas. These formulas form an integral part of the replacement policies which in turn depend to a great extent upon depreciation policies. In order to present the formulas clearly, it is necessary first to consider prevailing depreciation policies and their effect upon machine tool replacement policies.

Depreciation Policies

The word depreciation has many meanings. There are, according to Bonbright, four basic concepts on which all other definitions of the word are based.[10]

1. Decrease in value. This is simply the present value of an asset minus a future value of the asset. The difference in the value

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9. Hicks, E. M., "The Economics of Machine Replacement," Tool Engineer, September 3, 1951, p. 37.

10. Bonbright, J. C., Valuation of Property, New York, McGraw-Hill Company, 1937, Chapter X.

is the depreciation regardless of what has caused the reduction in value.

2. Amortized cost. This is the accounting concept by which the cost of an asset must be apportioned over its years of life. This is the concept that enters into income taxation.

3. Appraisal concept. This is the difference in value between an existing asset and a hypothetical new asset taken as a standard of comparison.

4. Impaired serviceableness. As machines become older they are not able to hold tolerances and produce finishes as well as when they were new. Their loss in operating capability is sometimes called depreciation.

The meaning of the word depreciation when used hereafter will be that given by amortized cost.

Most manufacturers use the Treasury Department allowable rate for purposes of accounting as well as for income tax purposes.[11] A summary by the Machinery and Allied Products Institute indicated that out of 182 companies answering a questionnaire, 84 percent used the same rates of depreciation for book and income tax purposes.[12]

The current depreciation policy of the United States was established by the Treasury Department in 1934 in T.D. 4422 and Mimeograph 4170. At that time probable lives of several thousand kinds of property

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11. Finney, Burnham, "Realistic Depreciation Rates," American Machinist, March 1, 1954, p. 118.

12. MAPI Survey, op. cit., p. 7.

were set up.[13] The manner in which depreciation was to be allowed is shown by the following statement from Bulletin F:

> A reasonable rate for depreciation is dependent not only on the prospective useful life of the property when acquired, but also on the particular conditions under which the property is used as reflected in the taxpayer's operating policy and the accounting policy followed with respect to repairs, maintenance replacements, charges to the capital asset account and to the depreciation reserve. If the useful life of the various assets shown hereafter could be determined precisely, which cannot be done, there still could not be established standard rates of depreciation unless there existed standard methods of operation and of accounting from which there could be no deviation.
>
> Being based on the usual experience of property owners, the probable useful lives shown herein for each kind or class of assets are predicated on a reasonable expense policy as to the cost of repairs and maintenance. Therefore, in the determination of the depreciation allowance in each case, due consideration should be given the maintenance and replacement policy of the taxpayer and the accounting practice regarding the same.
>
> The estimates of useful life set forth herein are for new properties only. In applying them, consideration should be given to salvage values, to that portion of the service life already expired, and to that portion of the cost previously recovered or recoverable through prior depreciation deductions or other allowances.
>
> It has been found that normal obsolescence is a very important factor in determining the useful life of property. The estimated useful lives shown herein include an allowance for normal obsolescence, but do not contain any provision for extraordinary obsolescence, such as is occasioned by revolutionary inventions, abnormal growth or development, radical economic changes, or other unpredictable factors which may force the retirement or other disposition of property prior to the termination of its normal useful life.

The effect of the new depreciation policy of the Internal Revenue Bureau upon industry was twofold; it reduced the depreciation rates on machine tools and it required that straight-line depreciation be used

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13. Bulletin F, Income Tax Depreciation and Obsolescence, Estimated Useful Lines and Depreciation Rates, Bureau of Internal Revenue, Revised, January, 1942.

for tax purposes.

The depreciation rate widely used for machine tools prior to 1934 was 10 percent; the present rate for the same types of machine tools is about 4 or 5 percent.[14] These lower rates were put into effect by giving the Internal Revenue Service the perogative of naming the depreciation rates on new equipment. If the manufacturer considered the rates improper, he had to prove them so.[15] Many smaller manufacturers had neither suitable records nor an adequate staff to handle the problem and so the rates contained in Bulletin F were used.

In calculating the depreciation rate, the Bureau of Internal Revenue requires that the taxpayer be able to prove that the straight-line depreciation rate used in his tax returns corresponds to the actual average service lives of the assets being depreciated.[16]

The straight-line depreciation rate for an asset is defined as:

$$\left[\frac{\text{First Cost - Estimated Salvage Value}}{\text{Estimated Service Life}}\right]\left[\frac{1}{\text{First Cost}}\right]$$

The straight-line method of depreciation does not give a realistic picture of the way productive equipment depreciates. Productive equipment normally loses two-thirds of its value to the owner in the first half of its service life.[17]

The straight-line method, therefore, does not allow enough

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14. Grant, E. L., Principles of Engineering Economy, Ronald Press, Third Edition, 1950, p. 182.

15. Finney, Burnham, op. cit., p. 115.

16. Grant, E. L., op. cit., p. 182.

17. Realistic Depreciation Policy - A Summary, Machinery and Allied Products Institute, Chicago, 1953.

depreciation on the new investment and it allows too much depreciation on the old investment. The true decrease in value can be better approximated by a declining balance method of depreciation. In this method, the annual depreciation charge is a fixed percentage of the remaining book value.[18]

The depreciation reserves that have been accruing on machines purchased before the post war inflation are inadequate for purchasing new machines. The degree of inadequacy of the depreciation reserves is indicated by a study by the Machinery and Allied Products Institute.

> When we undertake to adjust to a purchasing-power-equivalent basis the historical-cost depreciation now accruing on all business assets, we must first break down these assets by year of origin. The original-cost accrual on each year-of-origin group is then adjusted to yield the same purchasing power in 1953 dollars that it had in the dollars originally invested. The sum of these adjusted sub-accruals is of course the purchasing-power equivalent of the total original-cost depreciation.
>
> According to the estimates such an adjustment would add about 5 billion dollars to the annual depreciation of 14.2 billion dollars now being accrued on business assets, an increase of 35 percent.[19]

A number of suggestions have been offered to obtain a more flexible income tax depreciation policy for industry. The Machinery and Allied Products Institute recommends that tax payers be authorized to take a double-rate declining-balance write-off as an alternative to their present system and to blow up the original-cost book values to their equivalent dollars.[20] Finney recommends:

1. Allow manufacturers to set their own depreciation rates on new production equipment. But once a rate is adopted, it

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18. Grant, E. L., op. cit., p. 187.

19. Realistic Depreciation Policy - A Summary, p. 19.

20. Ibid, p. 35.

should not be changed. Maximum permissable write-off the first year: 50%.

2. Permit manufacturers to depreciate new equipment at a higher rate the first few years than later on. Either the declining-balance method could be used or an arbitrary deductible percentage specified, with a liberal percentage allowed.

3. Abandon the "useful life" theory as a basis for setting depreciation rates.

4. Eliminate Bulletin F of the Internal Revenue Service as a criterion for determining depreciation rates.

5. Put the burden of proof on the Internal Revenue Service for determining a tax payer to be wrong in his deductions for depreciation.[21]

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21. Finney, Burnham, op. cit., p. 115.

MACHINE TOOL REPLACEMENT FORMULAS

Annual Costs Method

Some authorities believe that in most cases it is advantageous to make replacement studies using annual costs.[22] The annual costs for each of the pieces of equipment under consideration are calculated and then compared. The equipment having the lowest annual cost is the most economical one to use neglecting intangible factors such as prestige, morale, etc. It must be recognized that at times these intangible factors may be the primary consideration in making a replacement decision, economic factors notwithstanding.[23]

The annual cost of a piece of equipment is made up of two parts, the capital cost and the operating cost.

Capital Cost. To determine the capital cost it is necessary to know the first cost of the equipment (c), and to estimate the service life of the equipment (n), an interest rate (i), and a salvage value at the end of the service life (s). The formula for capital cost is:

$$\text{Capital Cost} = (c - s)\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right] + si$$

The expression $\left[\frac{i\,(1+i)^n}{(1+i)^n - 1}\right]$ is called the capital recovery factor; it is derived in Appendix A. Tables of these factors can be

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22. Grant, E. L., op. cit., p. 358.

23. Ibid., pp. 21, 104.

found in most engineering economics books.[24]

The capital recovery factor is applied to the difference between the first cost of the equipment and the final salvage cost. This means that the capital that is depleted throughout the life of the investment must be recovered with a stated amount of interest. A straight interest rate is charged against the terminal salvage value since this may be thought of as a sum of money that will be paid back in a lump sum at some future date.

An approximate method which may be used to calculate the capital cost is called "straight line depreciation plus average interest." The straight line depreciation consists of the full depreciation of the equipment, that is first cost minus salvage, pro-rated over the life of the equipment.

$$\text{Straight line depreciation} = \frac{\text{first cost - salvage}}{\text{service life}}$$

The average interest charge may be calculated by noting the interest the first year is

$$(c - s)i + si$$

The interest the last year is

$$\frac{(c - s)}{n}i + si$$

The average is then

$$\left[(c - s) + \frac{c - s}{n}\right]\frac{i}{2} + si$$

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24. <u>Ibid.</u>, Appendix A.

or

$$(c - s)\left(\frac{1 + n}{n}\right)\frac{i}{2} + si$$

This is only an approximate solution. It is fairly accurate for low interest rates and a fairly short life. A chart of the errors involved may be found in most books on engineering economy.[25]

Operating Cost. The other cost to be included in the annual cost is the operating cost. This cost includes all expenses incident to the operation of the equipment such as labor, power, supplies, repairs, taxes, insurance, heat, light, etc.

Discussion. The annual cost method provides a simple, direct way to make a replacement analysis. The straight line depreciation plus average interest gives a reasonably correct solution if the capital recovery period is short, particularly where the estimated lives of the alternatives do not differ greatly. The exact method should be used for long estimated lives or for lives that differ widely.

One disadvantage in the use of this method is that no consideration is given explicitly to the superiority of future machines. The way that this problem is usually handled is to shorten the service life of the proposed machine. This technique demands a greater rate of return on the machine so that the machine may be paid for before it becomes obsolete.

Income taxes should be considered as a disbursement or series of disbursements in the same manner as maintenance costs, insurance, etc.[26]

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25. Thuesen, H. G., Engineering Economy, New York, Prentice-Hall, 1950, p. 93.

26. Grant, E. L., op. cit., p. 393.

The biggest difficulty is in estimating just exactly what the tax will be, especially when the tax is a graduated tax levied on net income. It is possible to choose an effective tax rate to simplify the calculations, but short cuts of this kind should be used with caution since errors will be introduced.[27]

Discounted Cash Flow Method[28]

This method uses the rate of return on an investment to determine the economic feasibility of making the investment. The acceptance or rejection of any particular proposed investment hinges upon whether or not the rate of return is greater than the company's cost of capital. If the rate of return is below this cost of capital, the project is rejected. If the rate of return is greater than the company's cost of capital, the project will be compared with other profitable projects to determine its acceptance. By this means, management has a file of profitable projects for consideration.

To use "discounted cash flow," it is necessary to estimate or calculate the future income that can be expected from the proposed investment and to estimate or calculate the future disbursements that will be incurred by the investment. When these two figures are available for each year of the estimated life of the investment, a net return for each year can be calculated. By use of a trial and error method it is possible to find an interest rate which discounts the net

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27. Ibid., p. 395.

28. Deon, Joel, "Measuring the Productivity of Capital," Harvard Business Review, January-February, 1954, vol. 32, No. 1, pp. 120-130.

returns of the investment down to the present cost of the project. The returns for the various years are converted into present worths at a number of different interest rates. When the sum of the present worths for a particular interest rate is equal to the initial investment, this interest rate is equal to the rate of return on the investment.

Discussion. The value of this method depends upon the accuracy with which future transactions can be estimated. The estimates of earnings from a proposal should be measured by the added savings that will accrue from making the investment as opposed to not making it. The estimated costs should be derived in the same way; that is, project costs should be unaffected by present overhead but should include changes in overhead caused by the investment. Nothing should be included in the estimated costs or earnings that would be present regardless of whether the proposal is accepted or rejected. These costs should cover the entire life of the investment since they can be expected to fluctuate from year to year and these fluctuations will affect the rate of return.

The correct basis for calculating the rate of return is the added outlay caused by the investment as opposed to rejecting it.

This rate of return method does not lend itself well to machine tool replacement studies since it is necessary to estimate returns and costs throughout the life of the machine. This is rather difficult to do with machines with service lives of ten years and above.

A desirable feature of this method is the manner in which projects can be arranged by degree of rate of return. While a number of

projects may be acceptable, the one that is the most profitable may be easily picked out.

Norton's Method

The method proposed by Paul T. Norton[29] is essentially an annual cost derivation. The costs involved for operating a piece of equipment to produce an expected annual output are tabulated under fixed charges and operating charges. The fixed charges are the capital costs, the operating charges are the operating costs.

The capital costs are calculated by taking the straight average of first cost and estimated salvage value. Norton believes it is usually accurate enough to assume no salvage; then the average investment is merely half of the first cost. The capital or fixed charges and the operating charges are summed for each piece of equipment. The sums are compared and the equipment with the lowest total is the most economical to use.

Norton has produced a legend to aid in listing the charges:

I = the investment or proposed equipment. For proposed equipment this should be the total cost in place ready to operate. For present equipment net realizable value (second hand or scrap, or value to owner for some other purpose), not book value.

A = annual % allowance for return on invested capital.

B = annual % allowance for taxes, insurance, etc.

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29. Norton, P. T., "Equipment Selection and Replacement," Manufacturer's Record, August, 1948, p. 48.

C = annual total cost (\$) of upkeep and maintenance.

D = annual % allowance for depreciation and obsolescence.

E = annual total cost (\$) of power, supplies, etc.

F = annual total cost (\$) of space allotted to machine.

L = annual total cost (\$) of direct labor.

M = annual total cost (\$) of material.

T = annual total cost (\$) of indirect expense.

Y = annual total fixed charges (\$) = I (A + B + D)

R = annual total charges of all kinds (\$) against machine for producing expected output.

= Y + C + E + F + M + L + T

Discussion. One advantage of the Norton Method is that complex computations are not necessary. The final analyst considers sums of money that are clearly labeled and the derivation of which is easily understood. Also the estimations are handled as approximations which is a realistic method of handling the usual replacement problem. Some factors such as salvage are neglected entirely. For most replacements the salvage value is low and may be neglected without serious error in the results of the analysis; however, there will be cases where the salvage will be appreciable and it will have to be considered. If exact figures are available for future expenditures, this method will not make the best use of these figures.

NMTBA Method

The National Machine Tool Builders' Association recommends the use of a formula which they have divided into three sections:

1. New machine savings in direct labor cost.
2. Cost of fringe benefits.
3. Depreciation allowances and federal income taxes.

The direct labor savings are calculated by using the difference in number of pieces per year that can be produced. If a limited number of pieces is desired, the saving in labor on this number of pieces should be used, not the number that it is possible to produce.

The cost of fringe benefits includes all other items that will be affected by the purchase of a new machine. The following is a check list of things according to NMTBA which should be considered:

Unemployment and old age benefits.

Paid vacations and holidays.

Group insurance, disability insurance, etc.

Sick pay, hospitalization, medical service.

Retirement plans, cafeteria losses, mutual aid.

Greater safety: lower rates for workman's compensation.

Lower maintenance costs.

Lower costs for direct labor.

Reduction of scrap.

Elimination of a production bottleneck.

Saving in floor space

Better operator morale.

Elimination of machines by combining operations.

Reduction of in-process inventory from faster flow of materials.

Faster assembly of the finished product.

Longer life and better operation of the product.

In this formula new equipment is written off in ten years. The NMTBA adopted the ten year amortization period in 1925 as a fairly good average of the length of life of a machine tool. Generally the Bureau of Internal Revenue requires a longer amortization period.[30] Therefore, the depreciation allowance that is income tax free is only a fraction of the total depreciation being used. The Association recommends making up the amount of the income tax on the unallowed depreciation by requiring a profit on the investment that will cancel out the income tax. This requirement is handled as an additional amount of capital to be recovered annually.

Discussion. This method compares operating costs but it does not compare capital costs. Capital costs are computed for the new machine only. The capital costs for the old machine are not considered. Since all of the costs for the present equipment are not considered, the method does not give a true picture but tends to slant the analysis in favor of the present equipment.

The rate of annual return on capital invested does not give a true picture for the entire life of the investment. The rate of return given by the method is true only for the first year; it is not the rate that can be expected over the life of the investment.

The depreciation period of ten years is not inflexible. The period

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30. Bulletin F., op. cit.

can be changed to suit the need. The tax rate may also be altered to suit the need of the particular company.

Rule of Thumb Method

There are a number of rules of thumb for determining when to replace a machine. One prevalent and popular method[31] is the short pay-off requirement; that is, the machine must pay for itself out of its savings in a short period of time. The periods of time run from one to five years.[32] The factors considered for determining whether a particular machine meets the pay-off requirement vary widely. For the example to be used here, the time to pay off will be calculated by:

$$\frac{\text{installed cost of new machine - salvage value of old machine}}{\text{yearly savings from new machine}}$$

The requirement for the maximum number of years to pay off was arbitrarily selected as three years.

Discussion. The three year pay off forces a complete amortization of the new machine in three years. If the old machine has a salvage value, the amount invested in the new machine is reduced by this amount. Thus the more the old machine is worth in salvage value, the smaller the savings have to be to meet the pay-off criterion. However, the higher the salvage value of the old machine, the less likely there is to be savings by using a new machine since salvage value is to some extent a measure of the ability of a machine to produce. The net result is that the salvage and the savings tend to cancel each other

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31. Grant, E. L., op. cit., p. 542.

32. MAPI Survey, op. cit.

out because as a machine gets older and has a decreasing salvage value, the savings that can be made by the purchase of a new machine go up. The effect of the short pay-off then is a protection of the old machine, the shorter the pay-off, the greater the protection.[33]

One reason for requiring a short pay-off or high rate of return is to protect a company against making unprofitable investment.[34] The shorter the pay-off required, the more profitable the investment. The error in reasoning here is that, while waiting for the savings to get high enough to warrant purchasing the new machine, the savings that would have accrued due to an earlier purchase are being lost.

The method used here for computing the rate of return is only one of many. A list of ten different ways that a rate of return may be calculated is contained in Terborgh's <u>Dynamic Equipment Policy</u>, pages 269-271.

MAPI Replacement Formula

The most recent research in machine tool replacement theory has been carried out by the Machinery and Allied Products Institute. The Institute is made up of a number of machine tool and equipment manufacturers and is headed by William J. Kelly, who is also president of a management counselling firm, William J. Kelly and Company, Chicago. Some of the member companies of the Institute are:

Brown and Sharpe Manufacturing Company, Providence, R. I.

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33. Terborgh, G., *op. cit.*, Chapter XII.

34. Thuesen, H. G., *op. cit.*, p. 185.

R. K. LeBlond Machine Tool Company, Cincinnati, Ohio

Mesta Machine Company, Pittsburgh, Pa.

Barber-Coleman Company, Rockford, Ill.

Link Belt Company, Chicago, Ill.

Allis Chalmers Mfg. Company, Milwaukee, Wisc.

E. W. Bliss Company, Toledo, Ohio

The Warner and Swasey Company, Cleveland, Ohio

Cincinnati Milling Machine Company, Cincinnati, Ohio

The research group at the Institute is under the direction of George Terborgh. The machine tool replacement formula was developed by this group and was published in two volumes written by Mr. Terborgh, Dynamic Equipment Policy, New York, McGraw Hill, 1949, and MAPI Replacement Manual, Chicago, Machinery and Allied Products Institute, 1950.

The first book, Dynamic Equipment Policy, explains the basic philosophy used in the approach to the problem.

> The present work . . . is an attempt to rethink fundamentally the underlying theory of equipment policy. . . . it is an attempt to integrate into this theory a recognition of the phenomenon of obsolescence, strangely ignored (as to future obsolescence at least) by existing theory. It is thus a piece of intellectual pioneering.[35]

Several new concepts in replacement studies are developed here, such as comparing successions of machines rather than single machines and determining the economic life of a machine by use of the ratio between the first cost of the machine and the rate at which a machine accumulates deterioration and obsolescence. A number of new terms are introduced in the formula such as challenger, defender, inferiority

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35. Terborgh, G., op. cit., p. vii.

gradient, adverse-minimum, to name a few.

Definition of Terms.

Challenger: The challenger is the proposed machine or process which is being compared with the present machine or process.

Defender: This is the machine or process presently in use.

Adverse Minimum: The adverse minimum is defined as the lowest time adjusted annual average of operating inferiority and capital cost. It is calculated for both the challenger and the defender. An explanation of how it is derived and used will be discussed later.

Operating Inferiority: The operating inferiority is the difference between the operating costs of the challenger and defender. These costs are made up of two broad factors; deterioration and obsolescence. Deterioration is caused by the wear and tear on the machine. It is reflected in downtime and maintenance costs. These cost factors follow a rather definite pattern and can be forecast with a fair degree of accuracy. Obsolescence is caused by new machines coming on the market with many new potentials built into them. These changes may be erratic but they are inevitable and they make the machine obsolete.

Inferiority Gradient: This is the rate, calculated on an annual basis, at which operating inferiority accumulates.

Capital Cost: The capital cost of the defender is made up of the decrease in salvage value during the next year plus the interest charge on the salvage value at the beginning of the year. The capital

cost of the challenger is contained in the formula which will be discussed later.

MAPI Method. The MAPI method consists of calculating an adverse minimum for the defender and an adverse minimum for the challenger. The adverse minimums are then compared. If the defender has the lower minimum, it should be kept in service. If the challenger has the lower minimum, it should replace the defender. For example, if the challenger's minimum is lower than the defender's minimum by an amount x dollars, and the defender is not replaced, then x dollars will be the cost of not replacing. In other words, the cost to the company of not replacing when replacement is indicated is the difference in the adverse minimums of the defender and challenger.

Computation of the Adverse Minimum for the Defender. The adverse minimum of the defender is the sum of the defender's capital cost and the defender's operating inferiority.

Capital Cost: The capital cost is made up of two parts, the prospective decrease in salvage value during the year and interest for the year on the present salvage value.

$$\text{Capital Cost} = (s_1 - s_2) + s_1 i$$

where s_1 = salvage value this year

s_2 = salvage value next year

i = interest rate

Operating Inferiority: The operating inferiority of the defender is determined by comparing the operating characteristics of the defender with the operating characteristics of the challenger. A complete check-list of the characteristics depends upon the type

of equipment and process under study. Some of the factors that should be considered are direct labor saving, supervisory and administrative costs, maintenance costs, cost of supplies, quality, capacity, power, property taxes, insurance, and floor space. The difference in the cost of the individual items is listed. The lists are then summed. The difference in these sums is the defender inferiority.

The adverse minimum of the defender is the sum of the defender's capital cost and the defender's inferiority.

Computation of Adverse Minimum for the Challenger. The adverse minimum of the challenger is made up of two parts, operating inferiority and capital cost. There are two methods that may be used to compute the adverse minimum of the challenger; one method yields an approximate adverse minimum, the other method yields an exact adverse minimum.

1. The Approximate Method: The method is only an approximation because it is based on the straight-line depreciation plus average interest technique for calculating capital recovery costs and because the inferiority gradient is only used to obtain a life average of operating inferiority. Straight-line depreciation plus average interest does not yield the true capital recovery cost and the life average of operating inferiority does not give the true operating cost.[36]

The life average of operating inferiority may be written as one-half of the inferiority gradient times the length of life minus one.

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36. Grant, E. L., op. cit., p. 99.

The minus one is required because the challenger is assumed to have no operating inferiority the first year. Straight-line depreciation is the acquisition cost minus the estimated salvage value at the end of the estimated service life divided by the estimated service life. Average interest may be written as the interest rate multiplied by one-half of the acquisition cost plus estimated salvage value at the end of the estimated service life. The life average of operating inferiority plus straight-line depreciation plus average interest yields the life-average of operating inferiority and capital cost.

$$u = \frac{g(n-1)}{2} + \frac{c-s}{n} + \frac{i(c+s)}{2}$$

u = life average of operating inferiority and capital cost.

g = inferiority gradient

n = estimated length of service life

c = installed cost of equipment

s = estimated salvage value at n years

i = interest rate

The adverse minimum is the lowest average cost of operating inferiority and capital cost. In order to satisfy this definition, it is necessary to find the values of g, n, c, i, and s that make u a minimum. If all future salvage values are taken as zero, u can be differentiated with respect to time n. If the derivative is set equal to zero, it will be possible to find relationships between c, g, and n that will make u a minimum. Taking s = 0

$$u = \frac{g(n-1)}{2} + \frac{c}{n} + \frac{i}{2}$$

Differentiating

$$\frac{du}{dn} = \frac{g}{2} - \frac{c}{n^2}$$

Setting the right side equal to zero

$$\frac{g}{2} = \frac{c}{n^2}$$

Then

$$g = \frac{2c}{n^2}$$

or

$$c = \frac{gn^2}{2}$$

or

$$n = \sqrt{\frac{2c}{g}}$$

Substituting these relationships back into the original equation in order,

$$u_{min} = \frac{c}{n}\left[2 - \frac{1}{n}\right] + \frac{i}{2}$$

or

$$u_{min} = \frac{g}{2}\left[2n - 1\right] + \frac{i}{2}$$

or

$$u_{min} = \sqrt{2cg} + \frac{ic - g}{2}$$

Any of these equations will yield the adverse minimum. However, the minimum derived will be only an approximation of the true adverse minimum.

2. The Exact Method: The same factors used in the approximate method are again used in the exact method but this time they are

handled differently. Instead of using straight-line depreciation plus average interest to calculate capital recovery costs, time adjusted uniform annual equivalent capital recovery costs are used. Instead of using life average of operating inferiority to calculate operating inferiority costs, time adjusted uniform annual equivalent operating inferiority costs are used.

To derive the adverse minimum of the challenger by the exact method, it is necessary to use present worth factors and capital recovery factors to obtain uniform annual equivalent costs. According to Grant, uniform equivalent costs are used:

> To compare non-uniform series of money disbursements where money has a time value, it is necessary some how to make them comparable. One way to do this is by reducing each to an equivalent uniform annual series of payments.[37]

Some engineering economics books contain tables of capital recovery factors and present worth factors.[38]

The factors are used in the following ways:

If a sum of money has been invested at an interest rate, the equivalent uniform annual return that should be received for an interest rate i and period of time n may be found by multiplying the investment by the capital recovery factor for the particular value of n and i. This sum is the same as the Capital Cost of the investment.

If a future payment is to be made, it is necessary to find out first its present worth and then apply the capital recovery factor

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37. *Ibid.*, p. 86.

38. Thuesen, H. G., *op. cit.*, pp. 481-491.

to the present worth figure. If there is to be more than one future payment, the present worth of each is found and then the capital recovery factor is applied to the cumulative present worth figure. This sum may be thought of as the Equivalent Uniform Annual Cost. The sum of the uniform annual cost and the capital cost yields the Total Equivalent Uniform Annual Cost.

In the exact method, the equivalent uniform annual cost is made up of the inferiority gradient with the proper present worth and capital recovery factors applied. These costs for various years are added to the capital costs for the same years. The sum of these costs is the total equivalent uniform annual cost. The sum normally decreases for a while, reaches a minimum, then rises. The key to correct equipment policy is, ". . . the policy that minimizes the time-adjusted sum or combined average of capital cost and operating inferiority."[39] That is, the minimum value of the total equivalent uniform annual cost is the adverse minimum and the number of years it takes to reach this minimum is the correct service life of the equipment.

The way in which the various factors are used to calculate uniform annual equivalents can be shown best by an example:

A proposed challenger has an acquisition cost of $10,000. The estimated salvage value decreases over the years, reaching $2,000 at ten years. The interest rate is ten percent. An inferiority gradient is estimated at $168 a year.

The equivalent uniform annual operating costs are calculated

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39. Terborgh, G., op. cit., p. 63.

using the inferiority gradient, the interest rate, present worth factors, and capital recovery factors. The procedure for handling the calculations is shown in Table 3.

The equivalent uniform annual capital costs are calculated using the salvage value, the interest rate, present worth factors, and capital recovery factors. The procedure for handling the calculations is shown in Table 4.

The sums of the equivalent annual capital costs and operating inferiority costs are contained in Table 5 and shown in Figure 1. The adverse minimum is given when this sum is a minimum. The year at which this minimum occurs is the year at which the machine should be retired.

The tabular method can be reduced to a formula or equation if some assumptions are made:

1. The challenger must accumulate operating inferiority at a constant rate.
2. The salvage value of the challenger must decrease according to a negative exponential law.

With these assumptions, the derivation of the formula in Appendix B is valid. In order to use the formula, it is necessary to know the acquisition cost of the challenger, the service life of the challenger, the salvage value at the end of the service life, and an interest rate.

The formula for the adverse minimum when the salvage value is expected to be zero throughout the life of the challenger is:

$$u_{min} = \frac{cni^2}{in + \frac{1}{(1+i)^{-n}} - 1} \qquad \text{(See Appendix B)}$$

Table 3

Computation of
Equivalent Uniform Annual Operating Inferiority Costs

Year	Inferiority Gradient	Present Worth Factors	Present Worth Operating Inferiority	Total Present Worth of Operating Inferiority	Capital Recovery Factors	Equivalent Uniform Annual Cost
1	0	.909			1.1	
2	168	.8264	138	138	.57619	80
3	336	.7513	252	391	.40211	157
4	504	.6830	344	735	.31547	232
5	672	.6290	422	1158	.26380	306
6	840	.5645	474	1632	.22961	375
7	1008	.5132	517	2149	.20541	442
8	1176	.4665	548	2698	.18744	506
9	1344	.4241	569	3268	.17364	568
10	1512	.3855	582	3851	.16275	627
11	1680	.3505	588	4439	.15396	684
12	1848	.3186	588	5028	.14676	738
13	2016	.2897	584	5612	.14078	790

Table 4

Computation of Equivalent Uniform Annual Capital Costs

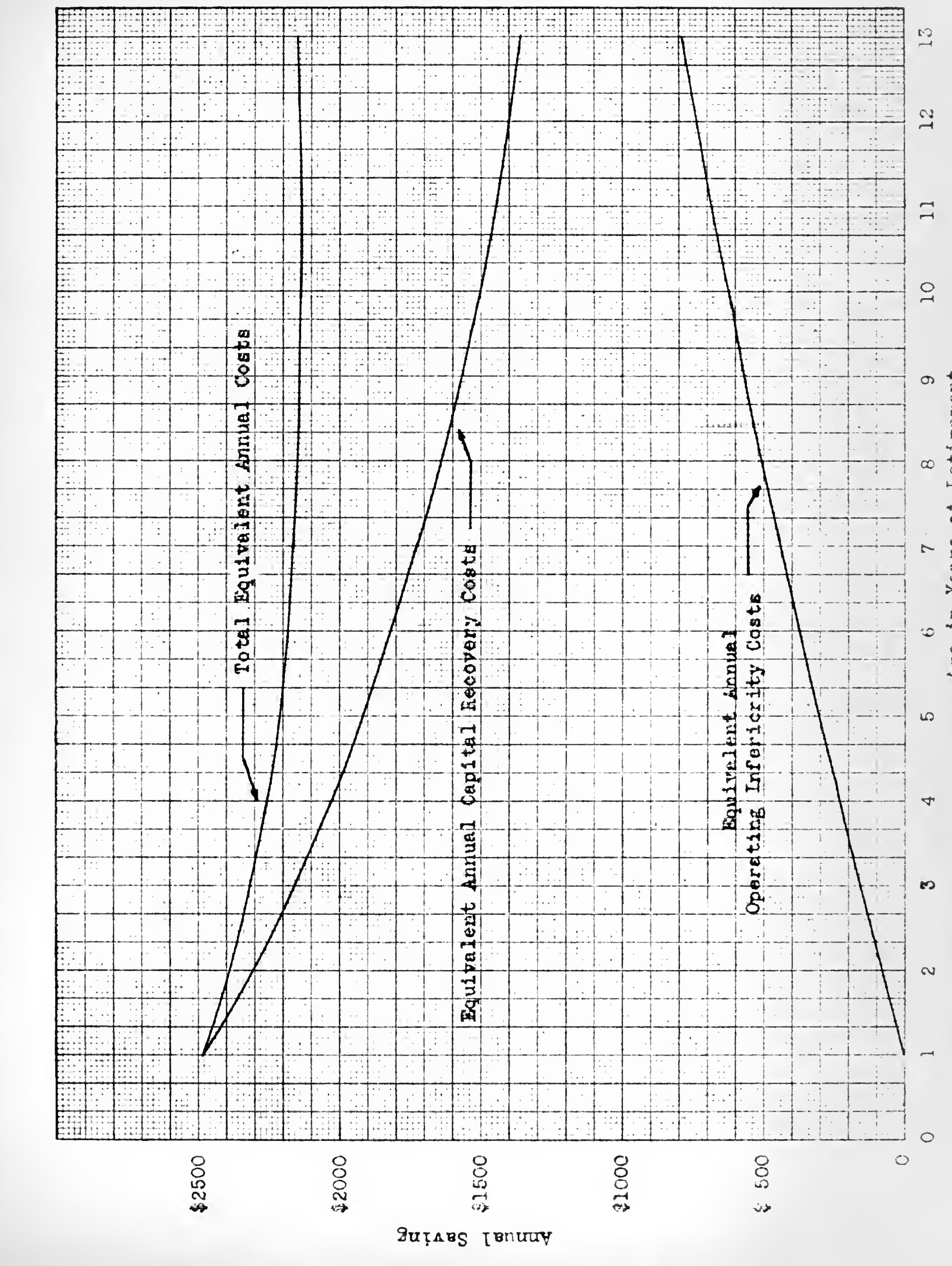
Year of Service	Salvage Value nth Year	Loss in Salvage	Interest on Salvage	Total Cost	Present Worth Factor	Present Worth of Costs	Total Present Worth of Costs	Capital Recovery Factor	Equivalent Uniform Annual Cost
0	10,000								
1	8513	1487	1000	2487	.909	2260	2260	1.1	2486
2	7247	1266	851.3	2117.3	.8264	1749	4010	.57619	2311
3	6169	1078	724.7	1802.7	.7513	1353	5364	.40211	2157
4	5252	917	616.9	1533.9	.6830	1047	6411	.31547	2028
5	4471	781	525.2	1306.2	.6290	821	7233	.26380	1908
6	3809	662	447.1	1109.1	.5645	671	7905	.22961	1815
7	3244	565	380.9	945.9	.5132	485	8390	.20541	1724
8	2759	485	324.4	809.4	.4665	377	8768	.18744	1644
9	2349	410	275.9	685.9	.4241	290	9059	.17364	1573
10	2000	349	234.9	583.9	.3855	225	9284	.16275	1511
11	1700	300	200.0	500.0	.3505	175	9459	.15396	1456
12	1450	250	170.0	420.0	.3186	133	9593	.14676	1408
13	1240	190	145.0	335.0	.2897	97	9690	.14078	1364

Table 5

Sum of Equivalent Uniform Annual
Capital Costs and Operating Inferiority

Year	Equivalent Uniform Annual Capital Cost	Equivalent Uniform Annual Operating Inferiority	Total Equivalent Uniform Annual Costs
1	2486	0	2486
2	2311	80	2391
3	2157	157	2314
4	2028	232	2260
5	1908	306	2214
6	1815	375	2190
7	1724	442	2166
8	1644	506	2150
9	1573	568	2141
10	1511	627	2138*
11	1456	684	2140
12	1408	738	2146
13	1364	790	2154

*Adverse minimum.


Total Equivalent Annual Costs
Equivalent Annual Capital Recovery Costs
Equivalent Annual
Operating Inferiority Costs
Annual Saving
$2500
$2000
$1500
$1000
$ 500
0
0
1
2
3
4
5
6
7
8
9
10
11
12
13
Age in Years at Retirement

When the salvage value at the end of the service life can be estimated, the formula is

$$u_{min} = \frac{in(ci+msp) - s(i+m)(1-p)}{in + p - 1} \qquad \text{(See Appendix B.)}$$

where c = acquisition cost of the challenger

s = salvage value at n years

n = service life

i = interest rate

p = present worth factor for the time and interest rate given, $(1+i)^{-n}$

$$m = \frac{2.3026}{n} (\log_{10} c - \log_{10} s)$$

The salvage value in the formula is given by

$$s = ce^{-mn}$$

For a given problem, the value of c and m will be known and it is then possible to calculate what the salvage value will be for each year of life. This is the way in which the salvage values in Table 4 were calculated. (See Appendix C.)

The exact formula for the salvage value case is rather unwieldy to use and so MAPI has put it in chart form. To use the chart it is necessary to enter with the ratio of terminal salvage value to acquisition cost of the challenger, and the service life of the challenger. (See Appendix B.)

<u>Discussion.</u>

Challenger Assumptions: In deriving the formulas for the challenger's adverse minimum, these assumptions are made:

1. The present challenger will accumulate operating

inferiority at a constant rate.

2. Any future challenger will have the same adverse minimum as the present challenger.

3. The salvage value, if used, will decrease according to the equation, $s = ce^{-mn}$.

Consequences of Challenger Assumptions: Because of the assumptions, the following conditions are implied:

1. The services of the challenger must have a perpetual need.

2. The acquisition cost of each future challenger must be the same as the acquisition cost of the present challenger.

3. Annual operating costs will increase at a constant rate for the present challenger and the same rate will apply to all succeeding challengers.

4. The superiority of future challengers will increase at a constant rate, this superiority rate will be evidenced by a stated reduction in the first year's operating costs.

5. The salvage value of the present challenger will decrease in a stated manner and all future challengers will have the same decreasing pattern.

If any of these conditions cannot be met by the proposed challenger, the formula cannot properly be used. If a perpetual need for the services of the proposed challenger is not anticipated or if future challengers are not expected to decrease in inferiority according to the gradient, the formula should not be used. Some other reasons for

not using the formula are fluctuating market conditions which affect the acquisition cost or the salvage value of the challenger, or the appearance on the market of a vastly superior challenger. The usual case in machine tools is a marked superiority in product every ten years.[40]

A simplifying assumption is also made for the defender; it is that the next year's sum of operating inferiority and capital cost is the adverse minimum of the defender succession. This assumption will be true if the operating inferiority is rising faster than the capital cost is falling. Usually when a machine is ready for a replacement study these conditions will prevail. In the cases where the next year's sum is not the lowest that will obtain, it is necessary to calculate defender capital costs and inferiority costs for several years to find the adverse minimum.

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40. Beach, Carl, Proceedings of Industrial Engineering Conference, May 13-14, Purdue University, 1953.

APPLICATION OF THE FORMULAS

A machine tool replacement formula should answer the question, "Will it pay to make the replacement?" It should also indicate how much will be saved or the return to be expected as a consequence of making the replacement.

This section is made up of three parts:

1. A problem is presented and then solved by each of the formulas.

2. The reasons for the differences in the answers yielded by the formulas are determined by comparing all of the formulas to the Annual Cost formula. The selection of the Annual Cost formula as a standard is arbitrary. Its selection does not mean that it is the best method to use for this problem or any other problem. The differences in the way factors such as interest, depreciation, etc., are considered in a formula affect the annual saving yielded by the formula. These differences between the Annual Cost method and the formula being compared with it are listed with the monetary value caused by the difference.

3. Errors in estimation will affect the answer yielded by a formula. To determine how much effect an error in estimation in this problem has on a particular formula, one factor is varied while holding all other factors constant. In this manner it is possible to see how much the result given by the formula increases or decreases, and hence how sensitive the formula is to a particular factor. The factors that are varied are the estimated service life, the estimated

salvage value, and the interest rate.

The Rule of Thumb method is not analyzed in this section since there are no estimations in it.

Machine Tool Replacement Problem

A new machine, costing $10,000 fully installed, is proposed to do the work of two present machines. The new machine is expected to have a service life of ten years and a salvage value at that time of $2000. The old machines are worth $1000 each; next year the salvage value of each is expected to be $800. The old machines can run for five more years, at which time they will need a major overhaul. Their salvage value at that time will be $500 each. It costs $5000 to operate the two old machines and $3000 to operate the new machine to get out the same amount of work. Property taxes and insurance are two percent per year. The interest rate is ten percent. Federal income taxes will not be considered in this problem since the MAPI formula does not allow for them and an equitable basis for comparison between all formulas is desired.

MAPI Formula

Calculations for Defender's Adverse Minimum:

	Defender	Challenger
Operating Costs		\$2000
Taxes and Insurance	\$160	
	160	2000

Defender Operating Inferiority = \$1840

Defender's Capital Cost:

Loss in salvage next year + Interest on present value

\$400 + \$200 = \$600

Defender's Adverse Minimum:

\$1840 + \$600 = \$2440

Calculations for Challenger's Adverse Minimum:

$$\text{Salvage Ratio} = \frac{\$2000}{\$10{,}000} = .20$$

Service Life = 10 years

Enter the MAPI chart with this information and get the factor = .11.

Adding the interest rate to this factor

.10 + .11 = .21

Applying this rate to the challenger's cost

\$10,000 x .21 = \$2100   Challenger's Adverse Minimum

Cost of not replacing:

\$2440 - \$2100 = \$340

Annual Cost Method

New Machine:

Capital Cost:

Straight line depreciation

$$\frac{10,000 - 2000}{10} = \$\ 800$$

Average Interest

$$8000 \times \frac{.10}{2} \times \frac{11}{10} + 2000 \times .10 = 640$$

Operating Costs	3000
Taxes and Insurance	200
	$4640

Old Machine:

Capital Recovery:

Depreciation $\frac{2000 - 1000}{5} = \$\ 200$

Average Interest

$$1000 \times \frac{.10}{2} \times \frac{6}{5} + 1000 \times .10 = 160$$

Operating Costs	5000
Taxes and Insurance	40
	$5400

Difference = $5400 - 4640 = $760

Annual Saving = $760

Norton Method

Old Machines		New Machine
\$2000	I	\$10,000
.10	A	.10
.02	B	.02
.20	D	.10
.32	ABD	.22
\$2000 x .32 = \$640	Y	.22 x \$5000 = \$1100
5000	C	3000
\$5640		\$4100

Difference = \$5640 - \$4100 = \$1540

Annual Saving = \$1540

Rule of Thumb Method

$$\text{Years to pay off} = \frac{\text{New investment - Salvage on old investment}}{\text{Savings}}$$

$$= \frac{\$10,000 - \$2000}{1840}$$

$$= 4.35 \text{ years}$$

$$\text{Rate of Return} = \frac{1}{4.35 \text{ years}}$$

$$= 23 \text{ percent}$$

NMTBA Method

	Old Machines	New Machine
Operating Costs	$5000	$3000
Taxes and Insurance	40	200
	$5040	$3200

Annual Operating Savings = $5040 - $3200 = $1840

$$\text{Capital Cost} = \frac{10,000}{10 \text{ years}} = 1000$$

Annual Saving $ 840

Discounted Cash Flow Method

In order to use this method it is necessary to estimate what the future net incomes will be. Since these figures are not available from the statement of the problem, it will be necessary to arbitrarily select net incomes for each year of service of the new machine.

The operating saving for this year will be taken as the operating cost of the new machine less insurance and property taxes, subtracted from the operating cost of the old machine less insurance and property taxes.

	New Machine	Old Machine
Operating Cost	\$3000	\$5000
Insurance and Property Taxes	200	40
	\$3200	\$5040

Operating saving this year = \$5040 - \$3200 = \$1840

To estimate the operating savings in the successive years, an operating inferiority gradient of \$100 will be used. This means that for each successive year the operating saving will be \$100 less than the year before.

At the end of the tenth year the estimated salvage value of \$2000 will be recovered.

Table 6

Discounted Cash Flow

Year	Operating Saving	Discounted Cash Flow 10%	Discounted Cash Flow 12%
1	$1840	$1670	$1643
2	1740	1440	1385
3	1640	1234	1170
4	1540	1050	980
5	1440	895	816
6	1340	757	678
7	1240	635	561
8	1140	532	460
9	1040	440	375
10	940	362	302
Salvage	2000	771	644
		$9786	$9014

By exterpolating, the rate of return is found to be 9.5 percent.

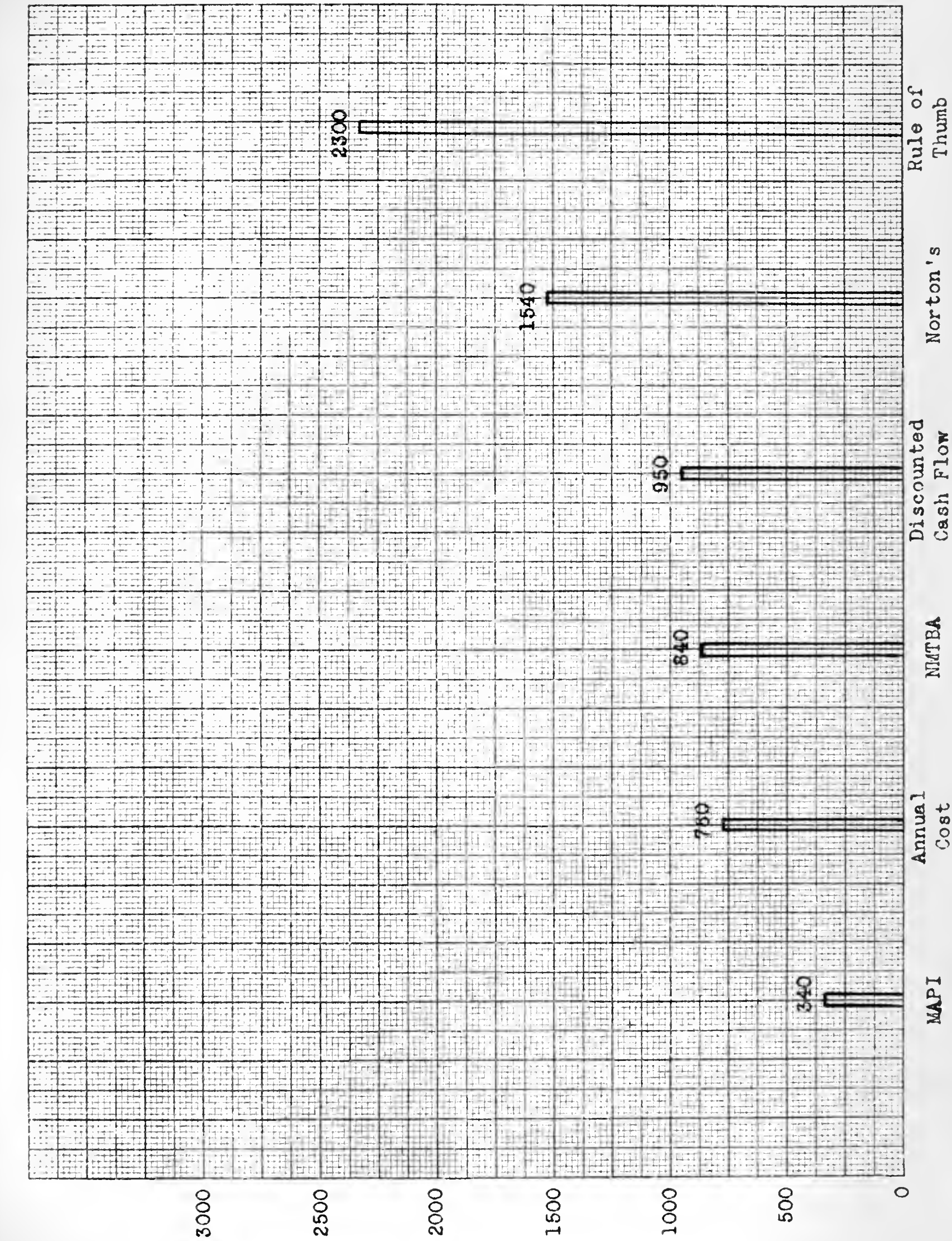


Fig. 2. Results of Problem

COMPARISON OF FORMULAS

Comparison of Annual Cost and NMTBA Method

Factors which will tend to make the NMTBA formula yield a higher yearly saving:

1. There is no requirement for a return with interest on the new machine. Difference =	$640

Factors which will tend to make the NMTBA formula yield a lower yearly saving:

1. The capital recovery for the old machine, both depreciation and interest, is not considered. Difference =	360
2. An allowance for the salvage value of the new machine at the end of its service life is not considered. Difference =	200
NMTBA method indicates a higher yearly saving of	$ 80

Comparison of Annual Cost and Norton's Method

Factors which will tend to make Norton's method yield a higher yearly saving:

	Difference
1. The interest charge on the old machine is not made by the average interest method with an an allowance for salvage value.	\$ 40
2. The depreciation rate does not allow for a salvage value on the old machine.	200
3. The interest charge on the new machine does not take into account the salvage value nor the $\frac{n+1}{n}$ factor in the average interest method.	140
4. The depreciation rate on the new machine does not take into account the salvage value and the rate is charged against only half of the installed cost of the new machine.	300
5. The property tax and insurance rate are charged against half of the installed cost of the new machine.	100
	\$780

Factors which will tend to make Norton's method yield a lower yearly saving:

None	0
Norton's method indicates higher yearly saving of	\$780

Comparison of Annual Cost and MAPI Method

Factors which will tend to make the MAPI formula yield a higher yearly saving:

	Difference
1. The capital recovery of the defender is calculated on the next year's decrease in salvage plus interest on the salvage value this year.	$240

Factors which will tend to make the MAPI formula yield a lower yearly saving:

1. The MAPI formula forecasts operating conditions into the future where as the annual cost method deals with operating costs this year.	
The difference caused by the future operating inferiority, approximately.	600
The difference caused by capital cost, approximately.	60
MAPI method indicates a lower yearly saving of	$420

Comparison of Annual Cost and Rule of Thumb Method

Factors which will tend to make the Rule of Thumb method yield higher yearly savings:

	Difference
1. The salvage value of the old machine is subtracted from the investment on the new machine.	$460
2. There is no capital recovery charge against the challenger.	1440
Factors which will tend to make the Rule of Thumb method yield lower yearly savings:	
1. There is no capital recovery charge against the defender.	360
Rule of Thumb indicates a higher yearly saving of	$1540

This higher yearly saving of $1540 is based on the assumption that a 23 percent return on the $10,000 investment means that the monetary yearly return is $2300. But if it is assumed that the 23 percent return is based on the $8000 investment, since $2000 will be realized on the sale of the defender, then the difference of $460 cannot be properly applied and the higher yearly saving will be $1080.

Comparison of the Annual Cost and Discounted Cash Flow Methods

Factors which will tend to make the Discounted Cash Flow method yield a higher yearly saving:

	Difference
1. There is no capital recovery charge against the defender.	$360
2. There is no interest charge against the challenger.	$640

Factors which will tend to make the Discounted Cash Flow method yield a lower yearly saving:

1. The decline in savings that will take place over the years is taken into account.	$810
The Discounted Cash Flow method indicates a higher yearly saving of	$190

EFFECT OF ERRORS IN ESTIMATION ON THE FORMULAS

Effect of Errors in Estimation on the Solution Yielded by the Annual Cost Method

1. The effect of an error in estimating the interest rate.

Table 7

Effect of Interest Rate on the Annual Cost Method

Interest Rate	Annual Cost		Annual Saving
	Challenger	Defender	
.05	$4320	$5320	$1000
.06	4384	5336	952
.07	4448	5352	904
.08	4512	5368	856
.09	4576	5384	808
.10	4640	5400	760
.11	4704	5416	712
.12	4768	5432	664
.13	4832	5448	616
.14	4896	5464	568
.15	4960	5480	520

2. Effect of an error in estimating the service life of the challenger.

Table 8

Effect of Service Life of the Challenger on the Annual Cost Method

Service Life in Years	Annual Cost		Annual Saving
	Challenger	Defender	
5	$5448	$5400	$ -48
6	5199	5400	201
7	5001	5400	399
8	4850	5400	550
9	4734	5400	666
10	4640	5400	760
11	4562	5400	838
12	4500	5400	900
13	4446	5400	954
14	4401	5400	999
15	4361	5400	1039

3. The effect on error in estimating the salvage value of the challenger.

Table 9

Effect of the Salvage Value of the Challenger on the Annual Cost Method

Salvage Value	Annual Cost		Annual Saving
	Challenger	Defender	
0	$4750	$5400	$650
500	4722	5400	678
1000	4695	5400	705
1500	4667	5400	733
2000	4640	5400	760
2500	4612	5400	788
3000	4585	5400	815

In the annual cost method the interest rate affects the indicated annual saving in a linear manner; a change in the interest rate of one percentage point yields a change of $48 in the indicated annual saving. The effect of a change in salvage value is also linear; a change of $500 in the salvage value yields a change of $27 in the indicated yearly saving. A change in service life does not effect the indicated annual saving in a linear way. The change in the indicated annual saving of $249 occurs between a 5 and 6 year life, while a change of only $49 occurs between a 14 and 15 year life.

Effect of Errors in Estimation on the Solution Yielded by Norton's Method

1. The effect of an error in estimating the interest rate.

Table 10

Effect of Interest Rate on Norton's Method

Interest Rate	Annual Cost		Annual Saving
	Challenger	Defender	
.05	$3850	$5540	$1690
.06	3900	5560	1660
.07	3950	5580	1630
.08	4000	5600	1600
.09	4050	5620	1570
.10	4100	5640	1540
.11	4150	5660	1510
.12	4200	5680	1480
.13	4250	5700	1450
.14	4300	5720	1420
.15	4350	5740	1390

2. The effect of an error in estimating the service life of the challenger.

Table 11

Effect of Service Life on Norton's Method

Service Life in Years	Annual Cost		Annual Saving
	Challenger	Defender	
5	$4600	$5640	$1040
6	4436	5640	1204
7	4313	5640	1327
8	4225	5640	1415
9	4155	5640	1485
10	4100	5640	1540
11	4055	5640	1585
12	4017	5640	1623
13	3985	5640	1655
14	3957	5640	1683
15	3933	5640	1707

3. Norton's method does not usually consider salvage value for the challenger, but if the machine does have a salvage value, then the indicated yearly saving will be in error.

Table 12

Effect of Challenger Salvage Value on Norton's Method

Salvage Value	Annual Cost		Annual Cost
	Challenger	Defender	
0	\$4100	\$5640	\$1540
500	4095	5640	1545
1000	4090	5640	1550
1500	4085	5640	1555
2000	4080	5640	1560
2500	4075	5640	1565
3000	4070	5640	1570

The indicated annual saving varies linearly with both the interest rate and the salvage value. A change of one percentage point in the interest rate yields an \$80 change in the indicated annual cost saving. A change of \$500 in salvage value yields a \$5 change in the indicated annual saving. A change in the service life does not affect the indicated annual saving in a linear way. A change of service life from 5 to 6 years yields a \$164 change in the indicated annual saving while a change of service life from 14 to 15 years yields a \$24 change.

Effect of Errors in Estimation on the Solution Yielded by the MAPI Formula

1. Effect of an error in estimating the interest rate.

Table 13

Effect of Interest Rate on MAPI Formula

Interest Rate	Adverse Minimum: Challenger	Adverse Minimum: Defender	Annual Saving
.05	$1600	$2340	$ 740
.06	1700	2360	660
.07	1800	2380	580
.08	1900	2400	500
.09	2000	2420	420
.10	2100	2440	340
.11	2200	2460	260
.12	2300	2480	180
.13	2400	2500	100
.14	2500	2520	20
.15	2600	2540	- 60

2. Effect of an error in estimating the salvage value at the end of the service life of the challenger.

Table 14

Effect of Salvage Value of the Challenger on MAPI Formula

Salvage Value	Adverse Minimum		Annual Saving
	Challenger	Defender	
$ 0	$2590	$2440	$ -150
500	2440	2440	000
1000	2300	2440	140
1500	2200	2440	240
2000	2100	2440	340
2500	2000	2440	440
3000	1900	2440	540

3. Effect of an error in estimating the service life of the challenger.

Table 15

Effect of Service Life of the Challenger on MAPI Formula

Service Life Years	Adverse Minimum		Annual Saving
	Challenger	Defender	
5	$3150	$2440	$ -710
6	2810	2440	-370
7	2560	2440	-120
8	2360	2440	80
9	2220	2440	220
10	2100	2440	340
11	1990	2440	460
12	1900	2440	540
13	1820	2440	620
14	1750	2440	690
15	1700	2440	740

The MAPI formula will yield a "correct" annual saving only if the estimates of its future life are made accurately. For this problem, an error of $500 in estimating the salvage value will change the indicated annual saving by $100. If the actual salvage value is $500 or less, the challenger should not replace the defender at this time. If the service life should actually be 7 years or less the defender should

not be replaced. The interest rate affects the indicated annual saving linearly; a one percent increase in the interest rate decreases the indicated annual saving by $80.

Effect of Error in Estimation on the Solution

Yielded by the NMTBA Method

The only estimation made in this method is the service life of the challenger.

Table 16

Effect of Service Life on the NMTBA Method

Service Life Years	Capital Cost	Annual Saving
5	$2000	$ -160
6	1667	173
7	1428	412
8	1250	590
9	1109	731
10	1000	840
11	909	939
12	824	1016
13	769	1071
14	714	1126
15	667	1173

The effect of an error in estimating service life is less for a challenger with long service life than it is for a challenger with a short service life.

Effect of Errors in Estimation on the Solution Yielded by the Discounted Cash Flow Method

The estimated service life as used in this method is closely allied with the annual savings that are expected each year because a saving for each year of service is explicitly indicated. A change in service life will therefore affect the annual savings expected. For this reason it seems appropriate to test for the effect of errors in estimating future savings rather than to test for errors in service life.

The errors in future savings are calculated on a straight percentage basis; that is, the savings for each year are considered to be a given percentage too high or too low. The salvage value at the end of the service life is considered to be in error by the same given percentage.

Table 17

Effect of Errors in Estimation on the Discounted Cash Flow Method

Error in Estimated Savings	Percentage Rate of Return
20% too high	5.3
10% too high	7.5
Correct	9.5
10% too low	11.4
20% too low	12.5

Discussion

The formulas for this problem yield annual savings which vary from \$340 to \$2300. This is a range of \$1960 in estimated savings for a machine costing only \$10,000. Obviously a replacement formula should never be used blindly. A company that intends to make a particular formula part of its machine tool replacement policy should analyze the formula carefully. The derivation of the formula should be fully understood so that the manner in which the formula utilizes all pertinent information is known.

Errors in estimating service life, terminal salvage value, and an acceptable interest rate will affect the answer yielded by a formula. The degree to which these errors in estimation affect the various formulas is not uniform.

The relative effect of the interest rate on three formulas is shown in Figure 3. For this problem, an error in estimating the interest rate has more effect on the MAPI formula result than on either Norton's or the Annual Cost Method. For all of the formulas there is an inverse relationship between the interest rate and the annual saving. This is because the interest charge is made before the annual saving is calculated.

The effect of service life on the answers yielded by the formulas is shown in Figure 4. The MAPI and NMTBA methods are about equally sensitive to changes in service life. The Annual Cost Method is less sensitive than the MAPI and NMTBA methods but is more sensitive than Norton's method.

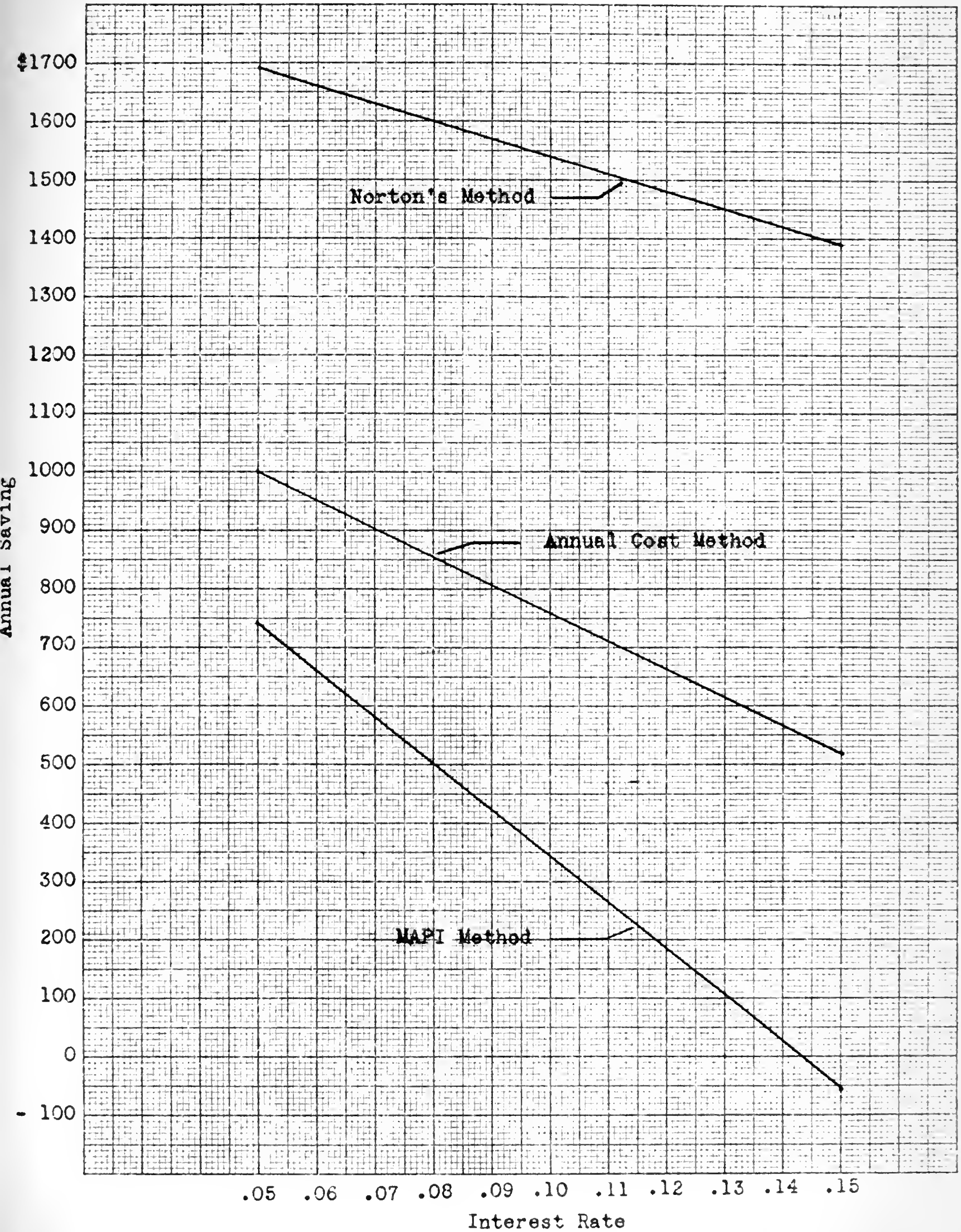


Fig. 3. Effect of Interest Rate on Annual Saving


Annual Saving
$1800
1600
1400
1200
1000
800
600
400
200
0
- 200
- 400
- 600
- 800
Years of Service Life
5
6
7
8
9
10
11
12
13
14
15
Norton's Method
NMTBA Method
Annual Cost Method
MAPI Method

The effect of salvage value on the answers yielded by the formulas is shown in Figure 5. The MAPI result is the most sensitive to errors in estimation of the terminal salvage value. The Norton result is practically unchanged by variations in the salvage value.

The MAPI formula's high degree of sensitivity to errors in estimation requires that care be exercised in making these estimates. It also indicates that the answer yielded is likely to be in error since estimates by definition are only approximations and are expected to be in error.

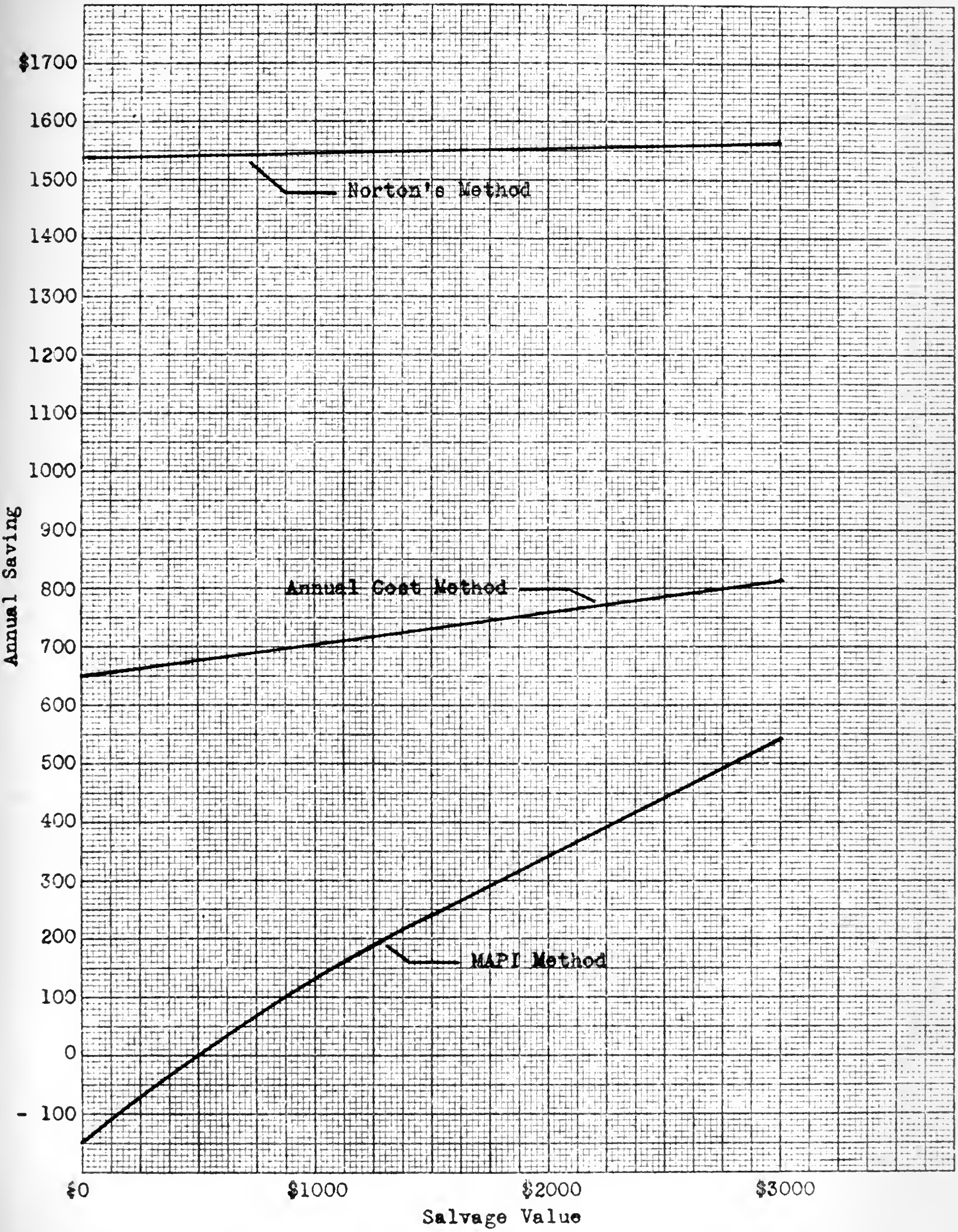


Fig. 5. Effect of Salvage Value on Annual Saving

CONCLUSIONS

The machine tool replacement formulas used by American Industry do not generally agree on the factors that should be used in making a replacement analysis, nor do they generally agree on how any particular factor should be used.

Most of the formulas compare alternatives to determine what the annual saving will be if the proposed machine is put in service.

For the problem considered in this thesis, the MAPI formula is the most sensitive to errors in estimation of the terminal salvage value, the MAPI and NMTBA formulas are more sensitive to errors in estimation of the service life, and the MAPI formula is the most sensitive to errors in estimation of the interest rate.

For the problem considered in this thesis, Norton's formula is the most insensitive to errors in estimated terminal salvage value, estimated service life, or estimated interest rate.

For the problem considered in this thesis, the MAPI formula yielded the lowest annual saving because it assigns a value to future operating losses. The Rule of Thumb method yielded the highest annual saving because it entertained no future charges.

Proposals for the Use of the Replacement Formulas

A machine tool replacement formula should be recognized as a tool of management and as any other tool it should be used only by a person who understands the limitations and recognizes the conditions for which it is suitable. One formula cannot yield the "correct" answer

to all replacement problems.

For replacement problems where the services of the challenger are expected to be in demand for a long time, the MAPI method yields realistic results.

The Discounted Cash Flow Method does not lend itself well to machine tool replacement problems. The difficulty in using the method arises when trying to estimate what the savings will be for future years. However, when these savings can be accurately estimated, the formula may be justifiably used.

The Rule of Thumb Method neglects many factors which the other formulas use. It should be used only for preliminary estimates and not as the final analysis.

The NMTBA, Annual Cost, and Norton's Method are suitable for replacement problems where the services of the machine are not expected to be required on a long term basis.

A company should conduct a continuous, or at least periodic, review of the equipment in the plant. This will serve the dual purpose of discovering new replacement opportunities and providing a check on the accuracy of the formula that is being used. If a formula indicated a yearly saving that is not being realized, then the reasons for this disparity should be investigated to find the cause. A formula that does not yield a correct yearly saving is not of much use to management and may do more harm than good.

APPENDIX A

DERIVATION OF PRESENT WORTH AND CAPITAL RECOVERY FACTOR

The present worth P of a future amount s in n years at interest rate i is given by:

$$P = s\left[\frac{1}{(1+i)^n}\right]$$

The equation is derived as follows: If P is invested at interest rate i, the interest for the first year is Pi and the total amount at the end of the first year is $P + iP = P(1 + i)$. The interest for the second year is $iP(1 + i)$ and the total amount at the end of the second year is $P(1 + i) + iP(1 + i) = P(1 + i)^2$. For n years the total amount is $P(1 + i)^n$.

Now let $s = P(1 + i)^n$. Then P is the principal which must be invested for n years at interest rate i to yield s. Or P is the present worth of a sum s paid n years hence at interest rate i.

The capital recovery factor yields the uniform annual end of the year payment, R, necessary to repay an investment, P, in n years at interest rate i.

$$R = P\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right]$$

The equation is derived as follows: If R is invested at the end of each year for n years, the total amount invested, s, will be equal to the sum of the compound amounts. The money invested the first year will earn interest for (n - 1) years; its amount will be

$R(1 + i)^{n-1}$. The second year's payment will be $R(1 + i)^{n-2}$. The payment at the last year will earn no interest. Then

$$s = R\left[1 + (1 + i) + (1 + i)^2 + \dots + (1 + i)^{n-1}\right]$$

Multiplying both sides by $(1 + i)$ and subtracting

$$is = R\left[(1 + i)^n - 1\right]$$

or

$$R = s\left[\frac{i}{(1 + i)^n - 1}\right]$$

noting that $s = P(1 + i)^n$ and substituting

$$R = P\left[\frac{i(1 + i)^n}{(1 + i)^n - 1}\right]$$

APPENDIX B

DERIVATION OF THE MAPI FORMULA

The MAPI formula can be derived in these steps:

1. Obtain expressions for operating inferiority and capital cost.
2. Obtain the uniform annual equivalent of the sum of the operating inferiority and capital cost.
3. Minimize the sum of the uniform annual equivalent of operating inferiority and capital cost.

<u>Obtaining an Expression for Operating Inferiority</u>: Operating inferiority is defined by the operating inferiority charge (inferiority gradient) g. The first charge is made at the end of the second year. The present worth of this charge is $g(1 + i)^{-2}$. The present worth of the charge for the next (third) year is $2g(1 + i)^{-3}$. Denoting the sum of the operating inferiorities by V gives:

$$V = g(1 + i)^{-2} + 2g(1 + i)^{-3} + \ldots + (n - 1)g(1 + i)^{-n} \tag{1}$$

The series can be summed in the following way: Multiply both sides by $(1 + i)^{-1}$

$$V(1 + i)^{-1} = g(1 + i)^{-3} + 2g(1 + i)^{-4} + \ldots + (n - 2)g(1 + i)^{-n} + (n - 1)g(1 + i)^{-n-1} \tag{2}$$

Subtracting (2) from (1),

$$V - V(1 + i)^{-1} = g(1 + i)^{-2} + g(1 + i)^{-3} + \ldots + g(1 + i)^{-n} - (n - 1)g(1 + i)^{-n-1} \tag{3}$$

Which can be reduced to:

$$V - V(1 + i)^{-1} = g(1 + i)^{-2} \left[1 + (1 + i)^{-1} + (1 + i)^{-2} + \ldots + (1 + i)^{-n+2}\right] - (n - 1)\, g(1 + i)^{-n-1} \quad (4)$$

The series in brackets on the right can be summed,

$$V - V(1 + i)^{-1} = g(1 + i)^{-2} \left[\frac{(1 + i)^{-n+1} - 1}{(1 + i)^{-1} - 1}\right] - (n - 1)\, g(1 + i)^{-n-1} \quad (5)$$

Solving for V,

$$V = \frac{g}{(1 + i)^2} \left[\frac{(1 + i)^{-n+1} - 1}{(1 + i)^{-1} - 1}\right] \frac{1}{1 - (1 + i)^{-1}} - \frac{(n - 1)\, g(1 + i)^{-n-1}}{1 - (1 + i)^{-1}} \quad (6)$$

Or,

$$V = \frac{g\left[(1 + i)^{-n+1} - 1\right]}{- i^2} - \frac{(n - 1)\, g(1 + i)^{-n-1}}{1 - (1 + i)^{-1}} \quad (7)$$

Then,

$$V = -\frac{1 + i}{i^3} \left\{g \left[(1 + i)^{-n+1} - 1 - (1 + i)^{-n} + (1 + i)^{-1}\right] + \left[(n - 1)\, i^2\, (1 + i)^{-n-1}\right]\right\} \quad (8)$$

And,

$$V = \frac{1}{-i^3} \left\{ g \left[(1 + i)^{-n+2} - i - (1 + i)^{-n+1} \right] + \left[(n - 1)\, i^2\, g (1 + i)^{-n} \right] \right\} \tag{9}$$

And,

$$V = \frac{1}{-i^3} \left\{ g \left[(1 + i)^{-n} (1 + 2i + i^2) - i - (1 + i)^{-n} (1 + i) \right] + \left[(n - 1)\, i^2\, g (1 + i)^{-n} \right] \right\} \tag{10}$$

Simplifying,

$$V = g \frac{\left[(1 + i)^{-n} - 1 + ni\,(1 + i)^{-n} \right]}{-i^2} \tag{11}$$

Finally,

$$V = \frac{g - g(1 + i)^{-n} (1 + ni)}{i^2} \tag{12}$$

The next step in the derivation is to obtain the uniform annual equivalent costs of both operating inferiority and capital cost. These uniform annual equivalent costs are obtained by multiplying the installed cost of the new machine c and the sum of the operating inferiority V by the capital recovery factor.

Let u = the uniform annual equivalent costs. Then,

$$u = (c + V) \left[\frac{i(1 + i)^n}{(1 + i)^n - 1} \right] \tag{13}$$

Substituting for V,

$$u = \left\{c + \left[\frac{g - g(1 + i)^{-n}\ (1 + ni)}{i^2}\right]\right\}\left[\frac{i(1 + i)^n}{(1 + i)^n - 1}\right] \tag{14}$$

Simplifying,

$$u = \frac{ci}{1 - (1 + i)^{-n}} + \frac{g - g(1 + i)^{-n}\ (1 + ni)}{i\left[1 - (1 + i)^{-n}\right]} \tag{15}$$

or,

$$u = \frac{ci^2 + g - g(1 + i)^{-n}\ (1 + ni)}{i\left[1 - (1 + i)^{-n}\right]} \tag{16}$$

To find the minimum value of u with respect to time n, the derivative will be taken and set equal to zero.

$$\frac{du}{dn} = \frac{i\left[1 - (1+i)^{-n}\right]\left[g(1+i)^{-n}(1+ni)\log(1+i) - ig(1+i)^{-n}\right]}{i^2\left[1 - (1+i)^{-n}\right]^2}$$

$$- \frac{\left[ci^2 + g - g(1+i)^{-n}\ (1 + ni)\right]\ i(1+i)^{-n}\ \log\ (1+i)}{i^2\left[1 - (1+i)^{-n}\right]^2} \tag{17}$$

Setting the right side equal to zero,

$$\begin{aligned} o = {} & ig(1 + i)^{-n}(1 + ni)\log(1 + i) - i^2g(1 + i)^{-n} \\ & - i(1 + i)^{-2n}g(1 + ni)\log(1 + i) + i^2g(1 + i)^{-2n} \\ & - ci^3(1 + i)^{-n}\log(1 + i) - gi(1 + i)^{-n}\log(1 + i) \\ & + ig(1 + i)^{-2n}\log(1 + i)\ (1 + ni) \end{aligned} \tag{18}$$

Simplifying,

$$\begin{aligned} o = {} & g\Big\{\left[i(1 + i)^{-n}\ (1 + ni)\ \log(1 + i)\right] - \left[i^2(1 + i)^{-n}\right] \\ & - \left[i(1 + i)^{-2n}\ (1 + ni)\ \log(1 + i)\right] + \left[i^2(1 + i)^{-2n}\right] \\ & - \left[i(1 + i)^{-n}\ \log(1 + i)\right] + \left[i(1 + i)^{-2n}\ \log(1 + i)\ (1 + ni)\right]\Big\} \\ & - ci^3(1 + i)^{-n}\ \log(1 + i) \end{aligned} \tag{19}$$

Then,

$$ci^2 \log(1 + i) = g\left[(1 + ni)\log(1 + i) - i + i(1 + i)^{-n}\right] - \log(1 + i) \qquad (20)$$

Or,

$$ci \log(1 + i) = g\left[n \log(1 + i) - 1 + (1 + i)^{-n}\right] \qquad (21)$$

The value of n in equation (21) is that value which yields the minimum value of u.

It is now possible to solve for u_{min} in terms of c, i, and n by solving for g in equation (21) and substituting this value of g into equation (16).

Solving for g in equation (21),

$$g = \frac{ci \log(1 + i)}{n \log(1 + i) - 1 + (1 + i)^{-n}} \qquad (22)$$

Then substituting in (16),

$$u_{min} = \frac{1}{i\left[1 - (1 + i)^{-n}\right]}\left\{ci^2 + \frac{ci \log(1 + i)}{n \log(1 + i) - 1 + (1 + i)^{-n}} - \frac{(i + 1)^{-n}(1 + ni)\, ci \log(1 + i)}{n \log(1 + i) - 1 + (1 + i)^{-n}}\right\} \qquad (23)$$

Simplifying,

$$u_{min} = c\left\{\frac{1}{\left[n \log(1 + i) - 1 + (1 + i)^{-n}\right]\left[1 - (1 + i)^{-n}\right]}\left(in \log(1 + i) - i + i(1 + i)^{-n} + \log(1 + i) - (1 + i)^{-n}(1 + ni)\log(1 + i)\right)\right\} \qquad (24)$$

Then,

$$u_{min} = c\left\{\frac{1}{\left[n \log(1 + i) - 1 + (1 + i)^{-n}\right]\left[1 - (1 + i)^{-n}\right]}\left(i\, n \log(1 + i)\left[1 - (1 + i)^{-n}\right] - i\left[1 - (1 + i)^{-n}\right] + \log(1 + i)\left[1 - (1 + i)^{-n}\right]\right)\right\} \qquad (25)$$

$$u_{min} = c\left[\frac{i\ n \log(1+i) - i + \log(1+i)}{n \log(1+i) - 1 + (1+i)^{-n}}\right] \tag{26}$$

If the approximation $\log(1 + i) = i$ is made,

Then,

$$u_{min} = \frac{c\ n\ i^2}{i\ n - 1 + (1+i)^{-n}} \tag{27}$$

Equation (27) is contained on page 72 of the <u>MAPI</u> <u>Replacement</u> <u>Manual</u>.

It is possible to get u_{min} in terms of n and g by solving (22) for c and substituting in (27).

$$c = \frac{g\left[n \log(1+i) - 1 + (1+i)^{-n}\right]}{i \log(1+i)} \tag{28}$$

$$u_{min} = \frac{g\left[n \log(1+i) - 1 + (1+i)^{-n}\right]}{\left[i \log(1+i)\right]} \quad \frac{\left[n\ i^2\right]}{\left[i\ n - 1 + (1+i)^{-n}\right]} \tag{29}$$

$$u_{min} = \frac{g\ n\left[i\ n - 1 + (1+i)^{-n}\right] i^2}{i^2\left[i\ n - 1 + (1+i)^{-n}\right]} \tag{30}$$

$$u_{min} = g\ n \tag{31}$$

The above derivations do not consider salvage value for the machine. In most cases there will be a salvage value and it will probably decrease from year to year. If the salvage value is assumed to decrease exponentially, then:

$$s = Sce^{-mn} \tag{32}$$

Where

s = salvage value after n years

S = salvage value factor

c = capital cost

n = number of years

m = exponential coefficient corresponding to the absolute value of the slope of the salvage value curve on semi-logarithmetic paper.

The present value of s can be ascertained by multiplying by the sinking fund factor.

$$\text{Present value of s} = Sce^{-mn} \quad \frac{i}{(1+i)^n - 1} \tag{33}$$

Substituting into (16) for a complete uniform annual equivalent cost,

$$u = \frac{ci^2 + g - g(1+i)^{-n}(1+ni)}{i\left[1 - (1+i)^{-n}\right]} + \frac{iSce^{-mn}}{\left[(1+i)^n - 1\right]} \tag{34}$$

$$u = \frac{ci^2 + g - g(1+i)^{-n}(1+ni) - i^2Sce^{-mn}(1+i)^{-n}}{i\left[1 - (1+i)^{-n}\right]} \tag{35}$$

To find the minimum value of u with respect to time:

$$\begin{aligned}\frac{du}{dn} = \frac{1}{i^2\left[1 - (1+i)^{-n}\right]^2} \Big\{ & i\left[1 - (1+i)^{-n}\right]\Big[g(1+i)^{-n}(1+ni)\log(1+i) \\ & - ig(1+i)^{-n} + mi^2Sce^{-mn}(1+i)^{-n} + i^2Sce^{-mn}\log(1+i)(1+i)^{-n}\Big] \\ & - \left[ci^2 + g - g(1+i)^{-n}(1+ni) - i^2Sce^{-mn}(1+i)^{-n}\right]\Big[i(1+i)^{-n} \\ & \log(1+i)\Big]\Big\}\end{aligned} \tag{36}$$

Putting the right hand side equal to zero,

$$\begin{aligned}o = & \left[1 - (1+i)^{-n}\right]\Big[g(1+i)^{-n}(1+ni)\log(1+i) - ig(1+i)^{-n} \\ & + mi^2Sce^{-mn}(1+i)^{-n} + i^2Sce^{-mn}\log(1+i)(1+i)^{-n}\Big] - \Big[ci^2 + g \\ & - g(1+i)^{-n} - ngi(1+i)^{-n} - i^2Sce^{-mn}(1+i)^{-n}\Big]\left[(1+i)^{-n}\log(1+i)\right]\end{aligned} \tag{37}$$

Collecting,

$$\begin{aligned} o = \; & g(1+ni)\log(1+i) - ig - g(1+i)^{-n}(1+ni)\log(1+i) \\ & + gi(1+i)^{-n} - g\log(1+i) + g(1+i)^{-n}\log(1+i)(1+ni) \\ & + i^2Sce^{-mn}\left[m + \log(1+i) - m(1+i)^{-n} - \log(1+i)(1+i)^{-n}\right. \\ & \left. + \log(1+i)(1+i)^{-n}\right] - ci^2\log(1+i) \end{aligned} \tag{38}$$

Then,

$$\begin{aligned} ci^2\log(1+i) = \; & g\left[ni\log(1+i) - i + i(1+i)^{-n}\right] + i^2Sce^{-mn} \\ & \left[m + \log(1+i) - m(1+i)^{-n}\right] \end{aligned} \tag{39}$$

and,

$$\begin{aligned} ci\log(1+i) = \; & g\left[n\log(1+i) - 1 + (1+i)^{-n}\right] \\ & + iSce^{-mn}\left[ni + \log(1+i) - m(1+i)^{-n}\right] \end{aligned} \tag{40}$$

The value of n in (40) is that value which yields the minimum value of u.

The value g can be eliminated by solving for g in (40) and substituting in (35).

$$g = \frac{ci\log(1+i) - iSce^{-mn}\left[m + \log(1+i) - m(1+i)^{-n}\right]}{n\log(1+i) - 1 + (1+i)^{-n}} \tag{41}$$

Then,

$$\begin{aligned} u_{min} = \; & \frac{ci}{\left[1 - (1+i)^{-n}\right]} \\ & + \frac{\dfrac{c\log(1+i) - Sce^{-mn}\left[m + \log(1+i) - m(1+i)^{-n}\right]}{n\log(1+i) - 1 + (1+i)^{-n}}}{\left[1 - (1+i)^{-n}\right]} \\ & - \frac{\left[(1+i)^{-n}(1+ni)\right]\dfrac{c\log(1+i) - Sce^{-mn}\left[m+\log(1+i) - m(1+i)^{-n}\right]}{n\log(1+i) - 1 + (1+i)^{-n}}}{\left[1 - (1+i)^{-n}\right]} \\ & - \frac{iSce^{-mn}(1+i)^{-n}}{1 - (1+i)^{-n}} \end{aligned} \tag{42}$$

Simplifying,

$$u_{min} = \frac{1}{\left[n \log(1+i) - 1 + (1+i)^{-n}\right]\left[1 - (1+i)^{-n}\right]} \Big\{ ci\left[n \log(1+i) - 1 + (1+i)^{-n}\right] + c \log(1+i) - Sce^{-mn}\left[m + \log(1+i) - m(1+i)^{-n}\right]$$
$$- \left[(1+i)^{-n}(1+ni)\right]\left\{c \log(1+i) - Sce^{-mn}\left[m + \log(1+i) - m(1+i)^{-n}\right]\right\}$$
$$- \left[iSce^{-mn}(1+i)^{-n}\right]\left[n \log(1+i) - 1 + (1+i)^{-n}\right]\Big\} \qquad (43)$$

Simplifying,

$$u_{min} = \frac{1}{\left[n \log(1+i) - 1 + (1+i)^{-n}\right]\left[1 - (1+i)^{-n}\right]} \Big\{ cin \log(1+i) - ci$$
$$+ ci(1+i)^{-n} + c \log(1+i) - Sce^{-mn} \log(1+i) - Sce^{-mn} m$$
$$+ Sce^{-mn} m(1+i)^{-n} - c \log(1+i)(1+i)^{-n}(1+ni)$$
$$+ Sce^{-mn} m(1+i)^{-n}(1+ni) + Sce^{-mn} \log(1+i)(1+i)^{-n}(1+ni)$$
$$- Sce^{-mn} m(1+i)^{-2n}(1+ni) - iSce^{-mn} n \log(1+i)(1+i)^{-n}$$
$$+ iSce^{-mn}(1+i)^{-n} - iSce^{-mn}(1+i)^{-2n}\Big\} \qquad (44)$$

Collecting,

$$u_{min} = \frac{1}{\left[n \log(1+i) - 1 + (1+i)^{-n}\right]\left[1 - (1+i)^{-n}\right]} \Big\{ - ci\left[1 - (1+i)^{-n}\right]$$
$$+ c \log(1+i)\left[1 - (1+i)^{-n}\right] - niSce^{-mn}\left[1 - (1+i)^{-n}\right]$$
$$- Sce^{-mn} \log(1+i)\left[1 - (1+i)^{-n}\right] + mSce^{-mn}(1+ni)(1+i)^{-n}$$
$$\left[1 - (1+i)^{-n}\right] + iSce^{-mn}(1+i)^{-n}\left[1 - (1+i)^{-n}\right]$$
$$+ cin \log(1+i)\left[1 - (1+i)^{-n}\right]\Big\} \qquad (45)$$

Simplifying,

$$u_{min} = \frac{1}{\left[n \log(1+i) - 1 + (1+i)^{-n}\right]} \Big\{ - ci + c \log(1+i) - mSce^{-mn}$$
$$- Sce^{-mn} \log(1+i) + mSce^{-mn}(1+ni)(1+i)^{-n}$$
$$+ iSce^{-mn}(1+i)^{-n} + cin \log(1+i)\Big\} \qquad (46)$$

Using the approximation $i = \log(1+i)$, and noting from equation 32 that $s = Sce^{-mn}$ where p = present worth factor $(1+i)^{-n}$,

Then,

$$u_{min} = \frac{- ci + ci - ms - si + msp\ (1+ni) + isp + ci^2 n}{ni - 1 + p} \quad \ldots (47)$$

$$u_{min} = \frac{in(ci + msp) - s(i + m)\ (1 - p)}{ni + p - 1} \quad \ldots (48)$$

Equation (48) appears on page 71 of the <u>MAPI Replacement Manual</u>. m in equation (48) corresponds to r in the MAPI manual.

Page 51 in the <u>MAPI Replacement Manual</u> contains a chart which may be used to compute the adverse minimum of the challenger. The chart is entered with the estimated service life of the challenger and the ratio of estimated salvage value to purchase price of the challenger. The relationship between these factors and the adverse minimum, u_{min}, can be shown.

Let R be the ratio of salvage vale to purchase price,

$$R = \frac{s}{c} \quad (49)$$

Substituting in equation (48),

$$u_{min} = \frac{in(ci + mcRp) - cR(i + m)\ (1 - p)}{ni + p - i} \quad (50)$$

i = the interest rate

c = acquisition cost

s = estimated salvage value

n = estimated service life

p = present worth factor for the time n and interest rate involved. $(1 + i)^{-n}$

$$m = \frac{2.3026}{n}\left(\log_{10} c - \log_{10} s\right)$$

R = ratio of estimated salvage value to acquisition cost.

APPENDIX C

MAPI CALCULATION OF SALVAGE VALUE

The salvage value is given by,

$$s = ce^{-mn}$$

c = acquisition cost, \$10,000

n = service life, 10 years

$$m = \frac{2.3026}{n} (\log_{10} c - \log_{10} s), \quad 0.16094$$

Then,

$$s = 10{,}000\ e^{-0.16094n}$$

Table 18

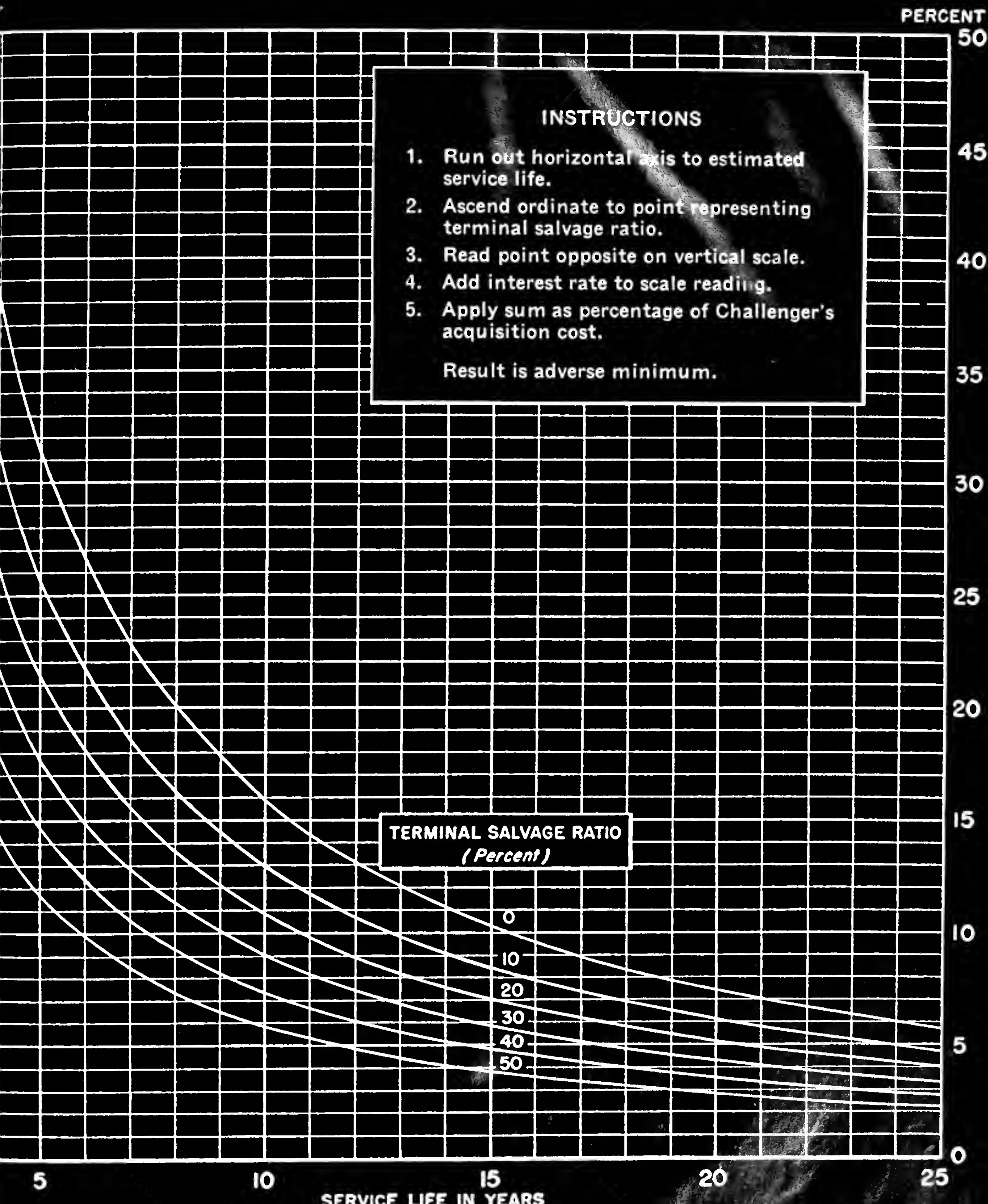
Salvage Values

n	s	n	s
1	\$8513	6	\$3809
2	7247	7	3244
3	6169	8	2759
4	5252	9	2349
5	4471	10	2000

APPENDIX D

REPLACEMENT FORMS

CHART FOR DERIVING CHALLENGER'S ADVERSE MINIMUM by the MAPI FORMULA



-EQUIPMENT ANALYSIS AND OPERATIONAL COMPARISON

ORK SHEET

SEE SUMMARIZING REPORT AND RECOMMENDATIONS (PAGE 1 OF 2).
SEE REVERSE SIDE FOR OTHER CALCULATIONS.

WILLIAM KELLY & COMPANY
ONE HUNDRED TWENTY SOUTH LA SALLE STREET
• CHICAGO 3 •

DATED ____________

NUMBER ____________

SUBJECT OF ANALYSIS ____________

ASSUMED RATE OF PRODUCTION ____________

A PRESENT EQUIPMENT—DEFENDER	B PROPOSED EQUIPMENT—CHALLENGER
A/B DESCRIPTION ____________	DESCRIPTION ____________
MACHINE NUMBER ____ DATE PURCHASED ____	MAKE AND SOURCE ____
LOCATION ____ INSTALLED COST $ ____	COST OF UNIT $ ____ INSTALLATION $ ____ TOTAL $ ____
DISPOSAL ____ SALVAGE OR CONVERSION VALUE $ ____	ESTIMATED PRIMARY SERVICE LIFE ____ ESTIMATED TERMINAL SALVAGE $ ____

OPERATIONAL NEXT-YEAR ADVANTAGES (DIFFERENCES)	A DEFENDER TOTAL	A DEFENDER ADVANTAGE	B CHALLENGER TOTAL	B CHALLENGER ADVANTAGE
INCOME ADVANTAGES				
A/B SUPERIORITY OF PRODUCT	$	$	$	$
INCREASED OUTPUT				
OTHER				
OPERATING COST ADVANTAGES				
DIRECT LABOR, INCL. OVERTIME & SHIFT PREMIUMS				
SET-UP TIME				
INDIRECT LABOR				
"FRINGE" LABOR COSTS				
ORDINARY MAINTENANCE				
SPECIAL REPAIRS				
TOOL COSTS				
SUPPLIES				
DEFECTIVE MATERIAL—REWORK				
SPOILAGE—SCRAP				
DOWNTIME—OUTAGE				
POWER CONSUMPTION				
FLOOR SPACE, IF USABLE				
PROPERTY TAXES AND INSURANCE				
SUB CONTRACT COSTS				
OTHER				
A/B TOTALS	$	$	$	$

DEFENDER OPERATING INFERIORITY (NET CHALLENGER ADVANTAGE) 32B-32A $ ____________

A ADVERSE MINIMUM—DEFENDER		B ADVERSE MINIMUM—CHALLENGER	
OPERATING INFERIORITY (LINE 33)	$	COST INSTALLED (TOTAL 8B)	$
SALVAGE VALUE LOSS, NEXT YEAR	$	PRIMARY SERVICE LIFE (9B)	
INTEREST @ ____ % (X LINE 9A)	$	TERMINAL SALVAGE VALUE (9B)	$
CAPITAL ADDITIONS, TOTAL $ ____		SALVAGE % (37B OF 35B)	
NEXT YEAR PRORATION	$	CHART ____ % INT. ____ % TOTAL	
INTEREST @ ____ % (X LINE 38A)	$	TOTAL % X COST INSTALLED (39B X 35B)	
TOTAL, OMITTING LINE 38 =		ANNUAL AVERAGE, PERIODIC CAPITAL ADDITIONS	$
ADVERSE MINIMUM	$	ADVERSE MINIMUM (40B + 41B)	$

NEXT YEAR GAIN FROM REPLACEMENT (42A MINUS 42B) $ ____________

ANALYSIS BY ____________ APPROVED ____________

SUMMARIZING REPORT AND RECOMMENDATION

WILLIAM KELLY & COMPANY
ONE HUNDRED TWENTY SOUTH LA SALLE STREET
• CHICAGO 3 •

DATED ______________________

NUMBER ______________________

RE-EQUIPMENT ANALYSIS AND OPERATIONAL COMPARISON

SUBJECT OF ANALYSIS:

RESULT—GAIN FROM REPLACEMENT:

REQUIRED INVESTMENT:
(Installed cost less salvage value of present equipment)

COMMENT AND SPECIAL EXPLANATION:

RECOMMENDATION:

APPROVAL AND COMMENT:

BY ______________________

DATE ______________________

WORK SHEET

The Cost of the New Machine is $................

	OLD MACHINE	NEW MACHINE
Direct labor cost per hour....................................	$________	$________
Fringe benefits per hour	$________	$________
Total hourly labor cost per hour............................	$________	$________
Divided by the number of parts produced per hour-units....	(________)	(________)
Gives us total labor cost per piece	$________	$________

•

The new machine produces .... pieces per day	
At $.............. per piece on old machines they would cost..................	$________
At $.............. per piece on new machines they would cost	$________
Labor savings per day..	$________
Annual savings, labor (40-hour week, 50 weeks per year)........................	$________
Estimated additional savings per year	$________
Total Annual Savings........................	$________

•

Desirable annual rate of recovery of capital invested in the new machine assuming it has a 10-year* profitable life (1/10 of cost)................................	$________
Amount recovered annually tax-free by 20-year* depreciation schedule (½ of above)	$________
Additional amount to be recovered annually out of profit..................	$________
Earnings required annually before taxes (at 38%) to recover above amount (above figure divided by .62)..	$________
Annual capital recovery required over the 10-year period; $.............. from depreciation plus $.............. from profit before taxes..................	$________

•

Total annual savings...	$________
Required annually for recovery of capital.....................................	$________
Annual net return on investment..	$________
Rate of annual return on capital invested; annual net return of $.............. divided by $.............., the cost of the new machine....................	________ %

** These periods vary, of course, depending on the nature of the machine and the product.*

NMTBA Replacement Work Sheet

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